MATH 1210 Assignment 2 Fall 2022 **Due date: October 24, 5:00 PM**

Attempt all questions and show all your work. Some or all questions will be marked.

1. Let $P(x) = x^4 + (2-i)x^3 + (1-2i)x^2 + (2-i)x - 2i$. Find all real values of a such that when the polynomial $P(x)$ is divided by $ix + a$ the remainder is 0.
2. Find all solutions of the cubic equation $P(x) = x^3 + x^2 - 2x - (3+\sqrt{3}) = 0$.
3. Consider the polynomial $P(x) = 4x^6 - 4x^5 + 11x^4 - 7x^3 + 5x^2 + 2x - 2$.
   (a) Use the Rational Root Theorem to find all possible rational zeros of $P(x)$.
   (b) Use Descartes' Rules of Signs to determine the number of possible positive real zeros and the number of possible negative real zeros of $P(x)$.
   (c) Use the Bounds Theorem to determine how large the absolute value of a root of $P(x)$ may be.
   (d) If $(\sqrt{2})i$ is a complex root of $P(x)$, find all zeros of $P(x)$ and express $P(x)$ as a product of linear factors.
4. Consider the polynomial $P(x) = 4x^4 + 5x^3 - 2x^2 + 5x - 6$.
   (a) Is $x + 2$ a factor of $P(x)$?
   (b) What are the possible rational roots of $P(x)$?
   (c) Show that if z is a complex number for which $|z| > \frac{5}{2}$ then z can not be a zero of $P(x)$.
   (d) What are the number of possible positive real zeros and the number of possible negative real zeros of $P(x)$?
   (e) Show that $P(x)$ has no zeros in the interval $[-5, -\frac{7}{3}]$.
   (f) Use your answers in parts **(a)**, **(c)** and **(d)** to improve the list of all possible rational zeros of $P(x)$ in part **(b)**.
   (g) Find all zeros of $P(x)$.
5. For each of the following, if it is true prove it and if it is not true give a specific example for which the statement is false.
   (a) If r_1 is a zero of polynomial $P_1(x)$ and r_2 is a zero of polynomial $P_2(x)$, then $r_1 - r_2$ is a zero of polynomial $P_1(x) - P_2(x)$.
   (b) If r is a zero of polynomial $P(x)$, then r^2 is a zero of polynomial $P(x^2)$.
   (c) If r is a zero of polynomial $P_1(x)$ of multiplicity k_1 and at the same time r is a zero of polynomial $P_2(x)$ of multiplicity k_2, then r is a zero of polynomial $P_1(x)P_2(x)$ of multiplicity $k_1 + k_2$.
6. Let $A = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 & -3 \\ 1 & 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 3 \\ -1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 6 \\ 1 & -1 \\ 0 & 5 \end{bmatrix}$.
   Evaluate each of the following expressions or explain why it is not defined.
   (a) $(5C + 2A)(7D - B^T)$;
   (b) $CA^T - 2BD$.
7. Let $A = \begin{bmatrix} -1 & -a \\ a & -2 \end{bmatrix}$; find all values of a for which $A^2 + 3A + 3aI_2 = \mathbf{0}$ where I_2 is the 2×2 identity matrix.
8. Let $\mathbf{u} = \langle 4, 1, 5 \rangle$, $\mathbf{v} = \langle 2, 1, 3 \rangle$ and $\mathbf{w} = \langle -a, 2, a \rangle$.
   (a) Find a vector of length 6 in the opposite direction of $2\mathbf{u} - 3\mathbf{v}$.
   (b) Find value(s) of a for which the angle between $2\mathbf{u} - 3\mathbf{v}$ and $\mathbf{w}$ is $\frac{2\pi}{3}$.