Attempt all questions and show all your work. Some or all questions will be marked.

1. For each of the following three systems of equations:
a Construct the associated augmented matrix and find its reduced row echelon form.
b Based on the answers in parts (a), for each system, state how many solutions it has. If a system has infinitely many solutions, write the set of all solutions in parametric form.

$$
\begin{align*}
& \text { (i) } \begin{array}{lll}
2 x+4 y-z=-1 & 2 x \quad+3 y=14
\end{array} \\
& \text { (i) } 2 x+2 y \quad=2 \quad \text { (ii) } \quad x+4 y=12 \\
& 3 x+6 y-z=0 \quad x \quad-y=3 \\
& \begin{array}{rrrrl}
2 x_{1} & +3 x_{2} & -5 x_{3} & +4 x_{4} & =3 \\
-x_{1} & +x_{2} & -3 x_{3} & -3 x_{4} & =-2 \\
& x_{3} & +2 x_{4} & =1
\end{array}  \tag{iii}\\
& x_{1}+2 x_{2}-4 x_{3} \quad+x_{4}=1
\end{align*}
$$

2. Find all basic solutions to the following homogeneous system.

$$
\begin{aligned}
2 x+3 y+5 z+2 w & =0 \\
3 y+z & =0 \\
4 x-6 y+2 z+4 w & =0 \\
3 x+3 y+7 z+3 w & =0
\end{aligned}
$$

3. Use Cramer's rule on the following system of equations to solve for $y$ without solving for $x, z$, or $w$.

$$
\begin{aligned}
& 3 x+y \quad+w=6 \\
& 2 x+z \quad=3 \\
& 4 y \quad+4 w=2 \\
& x+y \quad+2 w=2
\end{aligned}
$$

4. Let $A=\left[\begin{array}{ccc}1 & 0 & -1 \\ 0 & 1 & 4 \\ 5 & 1 & 0\end{array}\right]$ and $B=\left[\begin{array}{ccc}2 & 2 & 1 \\ 0 & 7 & 2 \\ 1 & -4 & -1\end{array}\right]$. Find the determinant of $A^{204} B^{97}-A^{202} B^{98}$.
5. Without expanding along any row or column, use the properties of determinants to find the determinant of the following matrix.

$$
\left[\begin{array}{ccccc}
1 & 0 & 0 & 0 & 3 \\
0 & -2 & 0 & 0 & 2 \\
0 & 0 & 0 & 4 & 0 \\
0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 4
\end{array}\right]
$$

6. Determine whether each of the following sets of vectors are linearly dependent or linearly independent.
a $\langle 2,3\rangle,\langle 5,2\rangle,\langle 1,7\rangle$
b $\langle 2,0,1\rangle,\langle 3,2,0\rangle,\langle 4,3,1\rangle$
c $\langle 3,0,3,5\rangle,\langle 4,2,2,4\rangle,\langle 0,-4,-1,2\rangle$
7. Find all values $a$ for which the set $\{\langle a-5,13,0\rangle,\langle-1, a-9,0\rangle,\langle 4,0, a-2\rangle\}$ is linearly dependent.
