**UNIVERSITY OF MANITOBA**
COURSE: MATH 1210
DATE & TIME: December 22, 2022, 1:30–3:30 PM
**Final** EXAMINATION
DURATION: 2 hours EXAMINER: Moghaddam, Penner, Shepelska

I understand that cheating is a serious offence:

**Signature** (*In Ink*): ______________________________

# INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 23 pages including this cover page and one blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

III. The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 90 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

V. Please do not call or e-mail your instructor to inquire about grades. They will be available shortly after they have been marked.

VI. If the QR codes on your exam paper are deliberately defaced, your exam may not be marked.

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[5] 1. Write the following sum in sigma notation.

$$\frac{4}{31} - \frac{7}{30} + \frac{10}{29} - \frac{13}{28} + \ldots + \frac{34}{21}$$

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[8] 2. Let $k$ be a complex number and $P(x) = 8x^3 - 4kx^2 + (2k+6)x + 3$. Find all values of $k$ for which $P(x)$ leaves remainder -1 when divided by $Q(x) = 2x - k$.

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3. Consider the points $P(1, -3, 4)$ and $Q(0, 2, 5)$ and the plane $\pi$ with equation $2x + y - 3z + 1 = 0$.

[4] (a) Find the vector of length 3 in the direction opposite to the one of $\overrightarrow{PQ}$.

[4] (b) Find an equation of the plane that passes through the point $P$ and is parallel to the plane $\pi$.

[4] (c) Find parametric equations of the line that passes through the point $Q$ and is perpendicular to the plane $\pi$.

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4. Consider the following homogeneous linear system:

$$\begin{array}{rrrrcl} 3x & +6y & +z & +8w & = & 0 \\ 4x & +8y & +z & +11w & = & 0 \\ 2x & +4y & +z & +5w & = & 0 \end{array}$$

[2] (a) Without doing any work to solve the system, how many solutions does this system have? You must provide an adequate reason for your answer.

[10] (b) Find the basic solutions to the system.

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[6] 5. Let $a \neq 1$ be a real number. Use Cramer's rule to solve the following system for $x$ **only**. No marks will be given for any other method.

$$\begin{array}{rrrcr} 2ax & -6y & & = & 2 \\ & 3y & +(a-1)z & = & -1 \\ (a+1)x & & +(a-1)z & = & -2 \end{array}$$

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[9] 6. Let $\mathbf{u_1} = \langle 1, 2, 0, 4\rangle$, $\mathbf{u_2} = \langle 0, 3, 1, 6\rangle$, and $\mathbf{u_3} = \langle 4, 5, -1, 10\rangle$. Find value(s) of $a$ for which the vector $\mathbf{v} = \langle a, -4, 2, a-3\rangle$ can be written as a linear combination of $\mathbf{u_1}$, $\mathbf{u_2}$, and $\mathbf{u_3}$.

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7. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ -5 & 5 & 1 \\ -3 & 4 & 0 \end{bmatrix}$.

[7] (a) Find $A^{-1}$.

[5] (b) Use part (a) to find the solution of the linear system $3A^T X = \begin{bmatrix} -6 \\ 0 \\ 3 \end{bmatrix}$.

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[7] 8. Consider the linear transformation $T(\langle x\,,\, y\rangle) = \langle x+y\,,\, x-y\rangle$ and let $\mathbf{u} = \langle 1\,,\,-2\rangle$. Find the vector $\mathbf{v} = \langle a\,,\, b\rangle$ such that $T(\mathbf{v}-2\mathbf{u}) = -3\,T(\mathbf{u}) - 2\mathbf{v}$.

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9. Let $A_{ab} = \begin{bmatrix} a^2 & a+b & -5 \\ 4 & b & 3 \\ -5 & -3a+2b & -a^2 \end{bmatrix}$.

[5] (a) Find values of $a$ and $b$ for which this matrix is symmetric.

[5] (b) For $a = b = 0$, find all eigenvalues of the matrix $A_{00} = \begin{bmatrix} 0 & 0 & -5 \\ 4 & 0 & 3 \\ -5 & 0 & 0 \end{bmatrix}$.

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[9] 10. Consider the matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. It is given that $|A - \lambda I| = -\lambda(\lambda - 2)^2$.

**You do not need to prove this step**. Use it to find two eigenvectors of $A$ which are orthogonal.

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BLANK PAGE FOR ROUGH WORK ( THIS PAGE WILL NOT BE MARKED.)