

I understand that cheating is a serious offence:

**Signature** (*In Ink*): \_\_\_\_\_

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## INSTRUCTIONS

- I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.
- II. This exam has a title page, 17 pages including this cover page and one blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.
- III. The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 60 points.
- IV. **Answer all questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but **CLEARLY INDICATE** that your work is continued.
- V. Please do not call or e-mail your instructor to inquire about grades. They will be available shortly after they have been marked.
- VI. If the QR codes on your exam paper are deliberately defaced, your exam may not be marked.

UNIVERSITY OF MANITOBA

COURSE: MATH 1210

**Midterm** EXAMINATION

DATE & TIME: October 27, 2022, 5:45–7:00 PM

DURATION: 75 minutes

EXAMINER: Moghaddam, Penner, Shepelska

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- [8] 1. Use mathematical induction to prove that

$$1 \cdot 1 + 2 \cdot 2 + 3 \cdot 4 + \dots + n \cdot 2^{n-1} = (n-1)2^n + 1$$

for all positive integers  $n$ .

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[6] 2. Use the identities

$$\sum_{j=1}^n j = \frac{n(n+1)}{2} \quad \text{and} \quad \sum_{j=1}^n j^2 = \frac{n(n+1)(2n+1)}{6}$$

to evaluate the following sum. You do not have to simplify your answer at the end.

$$\sum_{k=5}^{25} k(k-4)$$

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- [10]    3. Write the following complex number in Cartesian form. Simplify your answer as much as possible.

$$\frac{\left(\overline{\sqrt{3} + 3i}\right)^{20}}{3^{10}e^{\frac{5\pi i}{6}}}$$

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4. Consider the polynomial  $P(x) = x^4 - 6x^3 + 9x^2 - 6x + 8$ .

[3]      (a) Is  $x + i$  a factor of  $P(x)$ ?

[3]      (b) Use the Rational Roots Theorem to find all possible rational roots of  $P(x)$ .

$\Rightarrow$  Other parts of this question are in the next page

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- [4]      (c) Use Descartes' Rule of Signs to find the possible number of positive and the possible number of negative roots of  $P(x)$ .

- [5]      (d) Use the results from parts (a), (b), and (c) to find all roots of  $P(x)$ .

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[6]    5. Let  $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$ . Find the matrix  $X$  such that

$$(AB)^2 - AA^T - X = \mathbf{0}.$$

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6. Consider the vectors  $\mathbf{u} = \langle 1, 0, -1 \rangle$ ,  $\mathbf{v} = \langle \sqrt{2}, \sqrt{2}, 0 \rangle$ , the line  $\ell : x = 5 + 2t, y = 2 - t, z = t$  and the plane  $\Pi : 4x - y + 3z + 6 = 0$ .

[5]      (a) Find the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .

[4]      (b) Find the intersection point of the line  $\ell$  and the plane  $\Pi$ .

[6]      (c) Find an equation of the plane  $\Pi_1$  which is perpendicular to the plane  $\Pi$  and contains the line  $\ell$ .

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