MATH 1210 Assignment 1 Fall 2023 **Due date: October 3, 5:00 PM**

Attempt all questions and show all your work. Some or all questions will be marked.

1. Use mathematical induction to prove each of the following statements.

   (a) $1! \cdot 3 - 2! \cdot 4 + 3! \cdot 5 - 4! \cdot 6 + \ldots + (2n-1)!(2n+1) = (2n)! + 1$ for all positive integers n;

   (b) $\dfrac{4}{9} + \dfrac{8}{27} + \dfrac{12}{81} + \ldots + \dfrac{4n}{3^{n+1}} = 1 - \dfrac{2n+3}{3^{n+1}}$ for all positive integers n;

   (c) $7^n + 3n - 1$ is divisible by 9 for all integers $n \geq 2$.

2. Use the properties of sigma notation and the identities

$$\sum_{k=1}^{m} k = \frac{1}{2}\,[m(m+1)], \qquad \sum_{k=1}^{m} k^2 = \frac{1}{6}\,[m(m+1)(2m+1)], \qquad \sum_{k=1}^{m} k^3 = \frac{1}{4}\,[m^2(m+1)^2]$$

   to prove that

$$\sum_{k=n}^{4n-1} \big(2k - 3n\big) = 3n(2n-1).$$

   You are not allowed to use mathematical induction for this question.

3. Consider the sum $20 \cdot 23 + 19 \cdot 25 + 18 \cdot 27 + 17 \cdot 29 + \ldots + 7 \cdot 49$.

   (a) Write the sum in sigma notation.

   (b) Use the identities from the second question to evaluate the sum. You do not need to simplify your final answer to a single number: for example, expressions like $\dfrac{20 \cdot 21 \cdot 41}{6}$ do not require further simplification.

4. Write the following complex expressions in Cartesian from. Simplify as much as possible.

   (a) $\left(\dfrac{4i \cdot \overline{2-i}}{3-i}\right)^{15}$ (Hint: your answer should contain 2^{22}, and this power doesn't need to be simplified any further.)

   (b) $\dfrac{\big(2e^{\frac{\pi i}{12}} - i - 1\big)\big(2e^{\frac{\pi i}{12}} + i + 1\big)}{(1-i)^7}$

5. Find all complex solutions of the equation $(4z^2 - 2z + 1)(z^4 - 4i^3) = 0$. Write the roots in exponential form and use **principal values** of their arguments.