MATH 1210 Assignment 2 Fall 2023 **Due date: October 23, 5:00 PM**

Attempt all questions and show all your work. Some or all questions will be marked.

1. Let $P(x) = x^4 + (1+2i)x^3 + (4-3i)x^2 - 2x + 2a^3$. Find all real values of a such that when the polynomial $P(x)$ is divided by $x + ai$ the remainder is 0.

2. Let $P(x) = 2x^5 + 3x^4 - 2x - 3$.
   (a) Use Descartes' Rules of Signs to determine the number of possible positive real zeros and the number of possible negative real zeros of $P(x)$.
   (b) Is $x = 1$ is a zero of $P(x)$?
   (c) Use the Rational Root Theorem to find a list of possible rational roots of $P(x)$.
   (d) Use the Bounds Theorem to determine how large the absolute value of a root of $P(x)$ may be.
   (e) Use your answers in the previous parts to improve your list of all possible rational roots of $P(x)$ in part **(c)**.
   (f) Find all zeros of $P(x)$.

3. Consider the polynomial $P(x) = x^4 + x^3 + x^2 + 2x - 2$.
   (a) Show that $P(x)$ has no rational root.
   (b) Let z be a complex number such that $|z| \geq 3$. Prove that z can not be a root of $P(x)$.
   (c) If $(\sqrt{2})i$ is a complex root of $P(x)$, find all roots of $P(x)$ and express $P(x)$ as a product of linear factors.

4. Consider the matrices

$$A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 1 & 2 \\ -4 & 0 & 5 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \\ -2 & 3 \\ 4 & 2 \end{bmatrix}, \qquad D = \begin{bmatrix} 9 & -2 \\ 1 & 10 \end{bmatrix}.$$

   (a) For each of the following expressions, determine if it is defined. If yes, evaluate it. If no, explain why. (Note : In some cases, you can use rules for matrix algebra to simplify the expression, or even to determine the answer without calculation.)
      (i) $DD^T - D^T D$
      (ii) $BC - CB$
      (iii) $(AD)^T - D^T A^T$
      (iv) $3D^T + BC$
   (b) Find all values of a and b satisfying in the equation

$$A^4 - 8A^2 + D^T = \begin{bmatrix} 9 & 1 \\ -2 & -6 \end{bmatrix}.$$

5. Let $\mathbf{u} = \langle -7, 2, 1 \rangle$, $\mathbf{v} = \langle -9, 2, 1 \rangle$ and $\mathbf{w} = \langle a+1, a-1, 0 \rangle$.
   (a) Find a vector of length 7 in the opposite direction of $4\mathbf{u} - 3\mathbf{v}$.
   (b) Find value(s) of a for which the angle between $4\mathbf{u} - 3\mathbf{v}$ and $\mathbf{w}$ is $\dfrac{5\pi}{6}$.