UNIVERSITY OF MANITOBA
COURSE: MATH 1210
DATE & TIME: October 27, 2023, 5:45–7:00 PM
Midterm EXAMINATION
DURATION: 75 minutes EXAMINER: Kopotun, Moghaddam, Shepelska

I understand that cheating is a serious offence:

Signature (*In Ink*): ______________________________

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 17 pages including this cover page and one blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

III. The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 60 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

V. Please do not call or e-mail your instructor to inquire about grades. They will be available shortly after they have been marked.

VI. If the QR codes on your exam paper are deliberately defaced, your exam may not be marked.

BACK PAGE

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[8] 1. Use mathematical induction to prove that

$$\frac{5}{5\cdot 8}+\frac{5}{8\cdot 11}+\frac{5}{11\cdot 13}+\cdots+\frac{5}{(3n-1)(3n+2)}=\frac{n-1}{3n+2}$$

for every integer $n\geq 2$.

BACK PAGE

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[5] 2. Write the following sum in sigma notation. Your index of summation should start from 1.

$$\frac{2^{15}-1}{1\cdot 2}+\frac{2^{14}+1}{3\cdot 3}+\frac{2^{13}-1}{5\cdot 4}+\frac{2^{12}+1}{7\cdot 5}+\ldots+\frac{2^{7}-1}{17\cdot 10}$$

BACK PAGE

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[9] 3. Write the following expression in Cartesian from. Simplify your answer as much as possible.

$$\left[\frac{1-i}{i}\right]^{13}$$

BACK PAGE

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4. Let $P(x) = 4x^6 + 3x^4 + x^2 - 2x - 10$.

[5] (a) Use Descartes' Rules of Signs to prove that $P(x)$ has exactly 2 real zeros and 4 non real complex zeros.

[4] (b) Use the Rational Root Theorem to find a list of all possible rational roots of $P(x)$.

[4] (c) Use the Bounds Theorem to prove that $z = 3 + 4i$ can not be a root of $P(x)$.

BACK PAGE

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[7] 5. Consider the matrices $A = \begin{bmatrix} \frac{1}{2} & 1 \\ 0 & \frac{1}{2} \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -4 \\ 0 & 2 \end{bmatrix}$. Find a non identity matrix X satisfying the equation

$$2(AB)^T - B^T A^T = X^2 .$$

BACK PAGE

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[8] 6. Consider the two points $P(\lambda, 2, -1)$ and $Q(2, 1, -1)$, where λ is a real number. Find all values of λ (if any) such that the angle between $\overrightarrow{QP}$ and the vector $\mathbf{u} = \langle 0, 1, 1 \rangle$ is $\dfrac{\pi}{3}$.

BACK PAGE

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7. Consider the line

$$\ell: \quad x = 1 - t, \quad y = -2 + 4t, \quad z = 2 - 3t, \quad t \in \mathbf{R},$$

the plane

$$\Pi: \quad 3x + z + 1 = 0,$$

and the point $P(1, -1, 2)$.

[4] (a) Find the point Q of intersection of the line ℓ and the plane Π.

[6] (b) Find an equation of the plane Π_1 which contains the line ℓ and passes through the point P.

BACK PAGE

BLANK PAGE FOR ROUGH WORK (THIS PAGE WILL NOT BE MARKED.)