UNIVERSITY OF MANITOBA
COURSE: MATH 1700
DATE & TIME: December 15, 2024,
Final Examination
DURATION: 120 minutes EXAMINER: Various

Academic Integrity Contract I understand that cheating is a serious offence. "As members of the University Community, Students have an obligation to act with academic integrity. Any Student who engages in Academic Misconduct in relation to a University Matter will be subject to discipline." (2.4 - Student Academic Misconduct Procedure). :

Signature: ______________________________
(In Ink)

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 30 pages including this cover page. Please check that you have all the pages.

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 86 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. **Unjustified answers will receive little or no credit.** If you need more space, continue on the back of the page, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED. **Techniques from this course must be used.**

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[6] 1. Compute the (complex) fourth roots of -16. Put answers in Cartesian form.

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[6] 2. Let $P(x) = 6x^3 - 11x^2 + 2x + 8$.

(a) Use the rational root theorem to list all possible rational roots.

(b) Use that $P(-2/3) = 0$ to find all roots of the polynomial.

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3. Let $\vec{u} = \langle -1, 2, 3\rangle$, $\vec{v} = \langle 1, 3, -1\rangle$ and $\vec{w} = \langle 2, -1, 4\rangle$. Compute the following if they are defined. If they are not defined, provide a reason why not.

[4] (a) $(\vec{u} \times \vec{v}) \cdot \vec{w} + |\vec{w}|$

[2] (b) $(\vec{u} \times \vec{v}) + |\vec{w}|$

[4] (c) The value of $\cos\theta$ where θ is the angle between $\vec{v}$ and $\vec{w}$.

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[5] 4. Let P be the point $P(3,\ 2,\ 1)$ and Q be the point $Q(3, 2, -1)$.

Determine parametric equations of the line passing through P and perpendicular to both $\vec{PQ}$ and $\langle 1, 2, 3\rangle$.

[5] 5. Determine an equation of the plane containing both $P(3, 1, 2)$ and the line $x = 1 + t$, $y = -2 + 3t$, $z = -3 - t \quad t \in \mathbb{R}$.

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[6] 6. Use row reduction to find all solutions of the system of equations

$$\begin{aligned} x + y - z &= 6 \\ 2x - y + 3z &= 7 \\ -4x + 5y - 11z &= -9 \end{aligned}$$

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[5] 7. The augmented matrix $\left(\begin{array}{cccc|c} 1 & 2 & -3 & 4 & 0 \\ 2 & 5 & 0 & 6 & 0 \\ 3 & 7 & -3 & 10 & 0 \\ 1 & 3 & 3 & 2 & 0 \end{array}\right)$ of the system of equations

$$\begin{aligned} x + 2y - 3z + 4w &= 0 \\ 2x + 5y + 6w &= 0 \\ 3x + 7y - 3z + 10w &= 0 \\ x + 3y + 3z + 2w &= 0 \end{aligned}$$

has reduced row echelon form

$$\left(\begin{array}{cccc|c} 1 & 0 & -15 & 8 & 0 \\ 0 & 1 & 6 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

Find all basic solutions of the system.

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[5] 8. You are given that the three 2024×2024 matrices A, B, and C have determinants

$$\det(A) = 4, \qquad \det(B) = -2, \qquad \det(C) = 5.$$

What is the determinant of the matrix

$$(3AB^{-1}C^T)^T?$$

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9. You are given that the determinant of the coefficient matrix of the system of equations

$$\begin{aligned} 3x + 2y - z &= a \\ x + y + 2z &= b \\ x - 2y + 3z &= c \end{aligned}$$

where a, b and c are constants, is equal to 22.

[5] (a) Find the value for y in the solution set in terms of a, b and c.

[4] (b) Is the set of vectors $\{\langle 3,1,1\rangle, \langle 2,1,-2\rangle, \langle -1,2,3\rangle\}$ linearly independent or dependent? Explain your answer. (Hint: The vectors are the columns of the coefficient matrix)

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[7] 10. Let

$$A = \begin{bmatrix} 0 & 1 & -1 & 2 \\ -2 & 0 & 0 & 0 \\ 1 & -4 & 3 & -2 \\ 3 & -1 & 0 & -2 \end{bmatrix}.$$

You may use without proof that the determinant of A is 12. Without finding A^{-1}, use the adjoint method to find the entry in the second row and third column of A^{-1}. (You are NOT asked to find all entries of A^{-1}. No mark will be given for any other method.)

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[4] 11. Let $T : \mathbb{R}^3 \mapsto \mathbb{R}^3$ be a linear transformation such that $T(\langle 1,0,0\rangle) = \langle -1,2,3\rangle$, $T(\langle 1,1,0\rangle) = \langle -1,0,1\rangle$ and $T(\langle 1,1,1\rangle) = \langle 0,0,1\rangle$. Find the matrix associated with T.

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[5] 12. Suppose that $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ is a linear transformation that projects every vector onto the line $y = -x$. Find the matrix associated with T.

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[4] 13. Find all eigenvalues of the following matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 3 & 0 \\ 1 & 0 & -2 \end{bmatrix}.$$

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[6] 14. Let

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}.$$

It is known that the eigenvalues of A are $\lambda_1 = i$, $\lambda_2 = -i$ and $\lambda_3 = 0$ (you don't need to verify this fact). Find all eigenvectors of A corresponding to $\lambda_1 = i$.

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[3] 15. Explain (with all details) why there does not exist a 3×3 symmetric matrix with real entries that has three eigenvalues $\lambda_1 = 1$, $\lambda_2 = -1$, $\lambda_3 = 0$, and such that the eigenvectors corresponding to λ_1 are

$$t\langle 3, 2, 3\rangle, \quad t \neq 0,$$

the eigenvectors corresponding to λ_2 are

$$s\langle -1, -3, 3\rangle, \quad s \neq 0,$$

and the eigenvectors corresponding to λ_3 are

$$r\langle 3, -1, 0\rangle, \quad r \neq 0.$$

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