UNIVERSITY OF MANITOBA
COURSE: MATH 1210
DATE & TIME: ,
Term Test 4
DURATION: 25 minutes EXAMINER: various

Academic Integrity Contract I understand that cheating is a serious offence. "As members of the University Community, Students have an obligation to act with academic integrity. Any Student who engages in Academic Misconduct in relation to a University Matter will be subject to discipline." (2.4 - Student Academic Misconduct Procedure). :

Signature: ______________________________
(*In Ink*)

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 6 pages including this cover page. Please check that you have all the pages.

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 20 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. **Unjustified answers will receive little or no credit.** If you need more space, continue on the back of the page, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED. **Techniques from this course must be used.**

Use this space for scrap work or extra space if needed for a question. If you are using this for extra space, make sure it's clear on the original page that your work is continuing here.

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[8] 1. (a) Solve the system of equations and find basic solutions to the system.

$$\begin{aligned} x + 3y + 3z + 2w &= 0 \\ -2x - 6y - z + 6w &= 0 \end{aligned}$$

(b) Find basic solutions to the system.

Solution:

$$\begin{bmatrix} 1 & 3 & 3 & 2 & 0 \\ -2 & -6 & -1 & 6 & 0 \end{bmatrix} \Rightarrow^{R_2 \to R_2 + 2R_1} \begin{bmatrix} 1 & 3 & 3 & 2 & 0 \\ 0 & 0 & 5 & 10 & 0 \end{bmatrix} \Rightarrow^{R_2 \to R_2/5}$$
$$\begin{bmatrix} 1 & 3 & 3 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{bmatrix} \Rightarrow^{R_1 \to R_1 - 3R_2} \begin{bmatrix} 1 & 3 & 0 & -4 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{bmatrix}$$

Hence y and w are arbitrary and
$x = -3y + 4w$ and $z = -2w$. Thus

$$(x, y, z, w) = (-3y + 4w, y, -2w, w) = y(-3, 1, 0, 0) + w(4, 0, -2, 1)$$

Thus basic solutions are

$$\{(-3, 1, 0, 0), (4, 0, -2, 1)\}$$

Note, any vectors parallel to the above are also correct.

2. Suppose the following is the augmented matrix of a system of linear equations

$$\begin{bmatrix} 1 & 2 & 3 & | & 5 \\ 0 & 1 & 2 & | & 4 \\ 0 & 0 & a^2 - 6a & | & a - 6 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

With full explanation, determine which value(s) of a have the number of solutions to the system having:

[2] (a) Infinitely many solutions.

Solution:
You get infinitely many solutions when you have a consistent system with a column without a leading one. This happens when $a^2 - 6a = 0$ and $a = 6$. Thus $a = 6$.

[2] (b) No solutions

Solution: There is no solution when there is a 0 equals non-zero situation. Hence $a^2 - 6a = 0$ and $a \neq 6$. Thus $a = 0$.

[2] (c) Exactly one solution

Solution: There is exactly one solution when every non-augmented column has a leading one. Since every column has a leading one, this happens as long as the $(3, 3)$ entry is non-zero. Hence when $a \neq 0, 6$.

[6] 3. Consider the following system of equations below:

$$\begin{aligned} 6x + 3y + z - 5w &= 0 \\ 3x + 2y - z + w &= 0 \\ 4x - 3y + 7z - 9w &= 3 \\ x + 2y + 3z - 2w &= 0 \end{aligned}$$

Let A be the coefficient matrix. You can use that $\det(A) = -342$. Use Cramer's Rule to solve for y in the system above. Justify why Cramer's Rule is permitted to be used.

Solution:

Cramer's rule can be used since $\det(A) \neq 0$.

Along column 2

$$A_2 = \begin{vmatrix} 6 & 0 & 1 & -5 \\ 3 & 0 & -1 & 1 \\ 4 & 3 & 7 & -9 \\ 1 & 0 & 3 & -2 \end{vmatrix} = -0 + 0 - 3 \begin{vmatrix} 6 & 1 & -5 \\ 3 & -1 & 1 \\ 1 & 3 & -2 \end{vmatrix} + 0$$

Then along row 1

$$-3\begin{vmatrix}6 & 1 & -5\\ 3 & -1 & 1\\ 1 & 3 & -2\end{vmatrix} = -3\left(6\begin{vmatrix}-1 & 1\\ 3 & -2\end{vmatrix} - 1\begin{vmatrix}3 & 1\\ 1 & -2\end{vmatrix} + (-5)\begin{vmatrix}3 & -1\\ 1 & 3\end{vmatrix}\right)$$
$$= -3\left(6(-1) - (-7) + (-5)(10)\right)$$
$$= -3(-49)$$
$$= 147$$

Hence $w = \dfrac{147}{-342} = -\dfrac{49}{114}$.

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