

UNIVERSITY OF MANITOBA

COURSE: MATH 1210

DATE & TIME: ,

Term Test 4

DURATION: 25 minutes

EXAMINER: various

Academic Integrity Contract I understand that cheating is a serious offence. "As members of the University Community, Students have an obligation to act with academic integrity. Any Student who engages in Academic Misconduct in relation to a University Matter will be subject to discipline." (2.4 - Student Academic Misconduct Procedure). :

Signature: _____
(*In Ink*)

INSTRUCTIONS

- I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.
- II. This exam has a title page, 6 pages including this cover page. Please check that you have all the pages.
- III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 20 points.
- IV. **Answer all questions on the exam paper** in the space provided beneath the question. **Unjustified answers will receive little or no credit.** If you need more space, continue on the back of the page, **CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED. Techniques from this course must be used.**

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- [8] 1. (a) Solve the system of equations and find basic solutions to the system.

$$x - y + 3z + 4w = 0$$

$$2x - 10z + 6w = 0$$

- (b) Find basic solutions to the system.

Solution:

$$\begin{aligned} \begin{bmatrix} 1 & -1 & 3 & 4 & 0 \\ 2 & 0 & -10 & 6 & 0 \end{bmatrix} &\Rightarrow_{R_2 \rightarrow R_2 - 2R_1} \begin{bmatrix} 1 & -1 & 3 & 4 & 0 \\ 0 & 2 & -16 & -2 & 0 \end{bmatrix} \Rightarrow_{R_2 \rightarrow R_2/2} \\ \begin{bmatrix} 1 & -1 & 3 & 4 & 0 \\ 0 & 1 & -8 & -1 & 0 \end{bmatrix} &\Rightarrow_{R_1 \rightarrow R_1 + R_2} \begin{bmatrix} 1 & 0 & -5 & 3 & 0 \\ 0 & 1 & -8 & -1 & 0 \end{bmatrix} \end{aligned}$$

Hence y and w are arbitrary and

$x = 5z - 3w$ and $y = 8z + w$. Thus

$$(x, y, z, w) = (5z - 3w, 8z + w, z, w) = z(5, 8, 1, 0) + w(-3, 1, 0, 1)$$

Thus basic solutions are

$$\{(5, 8, 1, 0), (-3, 1, 0, 1)\}$$

Note, any vectors parallel to the above are also correct.

2. Suppose the following is the augmented matrix of a system of linear equations

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a^2 + 7 & a + 7 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

With full explanation, determine which value(s) of a have the number of solutions to the system having:

- [2] (a) Infinitely many solutions.

Solution:

You get infinitely many solutions when you have a consistent system with a column without a leading one. This happens when $a^2 + 7a = 0$ and $a = -7$. Thus $a = -7$.

- [2] (b) No solutions

Solution: There is no solution when there is a 0 equals non-zero situation. Hence $a^2 + 7a = 0$ and $a \neq -7$. Thus $a = 0$.

- [2] (c) Exactly one solution

Solution: There is exactly one solution when every non-augmented column has a leading one. Since every column has a leading one, this happens as long as the $(3, 3)$ entry is non-zero. Hence when $a \neq 0, -7$.

- [6] 3. Consider the following system of equations below:

$$6x + 3y + z - 5w = 0$$

$$3x + 2y - z + w = 2$$

$$4x - 3y + 7z - 9w = 0$$

$$x + 2y + 3z - 2w = 0$$

Let A be the coefficient matrix. You can use that $\det(A) = -342$. Use Cramer's Rule to solve for x in the system above. Justify why Cramer's Rule is permitted to be used.

Solution:

Cramer's rule can be used since $\det(A) \neq 0$.

Along column 1

$$A_1 = \begin{vmatrix} 0 & 3 & 1 & -5 \\ 2 & 2 & -1 & 1 \\ 0 & -3 & 7 & -9 \\ 0 & 2 & 3 & -2 \end{vmatrix} = 0 - 2 \begin{vmatrix} 3 & 1 & -5 \\ -3 & 7 & -9 \\ 2 & 3 & -2 \end{vmatrix} + 0 - 0$$

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Then along row 1

$$\begin{aligned} -2 \begin{vmatrix} 3 & 1 & -5 \\ -3 & 7 & -9 \\ 2 & 3 & -2 \end{vmatrix} &= -2 \left(3 \begin{vmatrix} 7 & -9 \\ 3 & -2 \end{vmatrix} - 1 \begin{vmatrix} -3 & -9 \\ 2 & -2 \end{vmatrix} + (-5) \begin{vmatrix} -3 & 7 \\ 2 & 3 \end{vmatrix} \right) \\ &= -2 (3(13) - (24) + (-5)(-23)) \\ &= -2(130) \\ &= -260 \end{aligned}$$

$$\text{Hence } w = \frac{-260}{-342} = \frac{130}{171}.$$

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