EXAMINER: various

Academic Integrity Contract I understand that cheating is a serious offence. "As members of the University Community, Students have an obligation to act with academic integrity. Any Student who engages in Academic Misconduct in relation to a University Matter will be subject to discipline." (2.4 - Student Academic Misconduct Procedure). :

Signature:

(In Ink)

INSTRUCTIONS

- I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.
- II. This exam has a title page, 16 pages including this cover page. Please check that you have all the pages.
- III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 57 points.
- IV. Answer all questions on the exam paper in the space provided beneath the question. Unjustified answers will receive little or no credit. If you need more space, continue on the back of the page, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED. Techniques from this course must be used.

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[8] 1. Use the principle of mathematical induction to show that

$$1 \cdot 3 + 2 \cdot 4 + \dots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$$

for all $n \ge 1$.

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[3] 2. Let $S_n = 1 \cdot 3 + 2 \cdot 4 + \cdots + n(n+2)$. Write S_n in terms of sigma notation.

[5] 3. Compute $\frac{4-3i}{2+i} + \overline{(3+4i)} \cdot (2i)$. The answer must be in Cartesian form.

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[6] 4. Compute $(\sqrt{3} - i)^{14}$. Write your answer in polar form using the principal value of the argument.

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5. Let $P(x) = 8x^3 + 2x^2 + x + 3$.

[3] (a) Using Descartes Rules of Signs, determine the number of possible positive roots and the number of possible negative roots.

[2] (b) Using the Bounds Theorem, determine the bound of the modulus of the roots.

[5] (c) It is given that P(-3/4) = 0. Find all roots of P(x).

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6. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -2 \\ 2 & 3 \\ 4 & 2 \end{bmatrix}$. Compute the following if they are defined. If they are not defined, explain why. Be specific.

[3] (a) AC - 3B

[3] (b) (CA)B

[3] (c) $CA + I_3$

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[5] 7. For the points $P(\lambda, 2, -1)$, Q(2, 1, -1) and R(-1, 1, 2), where λ is a real number, either find all of λ such that $\angle PQR = \pi/3$ or 60°, or show that such λ do not exist.

8. Suppose that we are given the point P(0, 1, 2), the plane

$$\Pi: y + 2z - 1 = 0$$

and the line

$$l: x = 1 - t, y = -2, z = 2 - 3t, t \in \mathbb{R}.$$

[3] (a) Find all points of intersection of the line l and the plane Π , or show that such points do not exist.

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Recall Π : y + 2z - 1 = 0 and l: x = 1 - t, y = -2, z = 2 - 3t, $t \in \mathbb{R}$.

[4] (b) Find an equation of the plane Π_1 which contains the line l and passes through the point P.

[4] (c) Find parametric and symmetric (if possible) equations for the line l_1 which is perpendicular to the plane Π and passes through the point *P*.