Instructions: Solve each of the following systems by using Gauss-Jordan elimination.

1.
$$-2x_1 - 2x_2 + 2x_3 = -6$$
$$4x_1 - x_2 + x_3 = 32$$
$$x_1 - x_2 - x_3 = 1$$
$$-3x_1 - 4x_2 + 3x_3 = -10$$

2.
$$x_1 - 3x_2 = 0$$
$$-2x_1 + 3x_2 = 3$$
$$-4x_2 = 4$$
$$2x_1 + 4x_2 = -10$$
$$4x_1 - x_2 = -11$$

3.
$$-x_1 + 2x_2 + 3x_3 = -36$$
$$-x_1 - 4x_2 - 2x_3 - x_4 + 4x_5 = 2$$
$$2x_3 + 2x_4 = -28$$

4.
$$-x_1 + 2x_3 + 3x_4 - 2x_5 - 2x_6 = 19$$
$$-2x_1 - 2x_2 - 3x_3 - 4x_4 + 3x_5 = -18$$
$$-3x_1 + 2x_2 - x_4 - x_6 = 5$$

5.
$$2x_1 - 2x_2 + 2x_3 + 4x_4 = -48$$
$$-4x_1 - 4x_2 - 2x_3 - 2x_4 = -22$$
$$3x_1 + 2x_3 - 2x_4 = 17$$
$$-x_1 - x_3 + 3x_4 = -26$$

6.
$$-3x_1 + 2x_2 + 2x_3 + 3x_4 = 3$$
$$x_1 + 2x_2 - 4x_4 = -12$$
$$2x_1 + 3x_2 - 3x_3 + 3x_4 = -77$$

7.
$$-4x_1 - 3x_2 - x_3 + 2x_4 + 4x_5 - 2x_6 = 71$$
$$-3x_1 - 4x_3 + 3x_4 + 2x_5 + 4x_6 = 69$$
$$3x_1 + 2x_2 + x_3 + 3x_4 = 9$$

8.
$$-4x_1 - 4x_2 - x_3 - 2x_4 + 3x_5 - 3x_6 = 18$$
$$-4x_1 - x_2 + x_3 + 4x_4 - 2x_5 + x_6 = 0$$

9.
$$-x_1 - 3x_2 - 2x_3 - 2x_4 = 36$$
$$2x_1 + 4x_2 - 3x_3 + 3x_4 = -7$$
$$-x_1 - 2x_2 - 4x_3 + 2x_4 = 30$$

10.
$$x_1 + 3x_2 - 2x_3 = 38$$
$$3x_1 - 3x_2 - 2x_3 = -38$$
$$-x_1 - x_2 - 4x_3 = 30$$
$$-3x_1 + 3x_2 = 54$$
$$-4x_1 + 3x_3 = 8$$

11.
$$2x_2 - x_3 = 8$$
$$3x_1 - 3x_2 + x_3 = -25$$
$$-x_1 + 4x_2 + 4x_3 = -3$$
$$-x_2 - 3x_3 = 10$$

12.
$$-3x_1 + 4x_2 - 4x_4 - 2x_5 = 21$$
$$2x_1 - 4x_2 + 4x_3 + 4x_4 + 3x_5 = 5$$
$$3x_1 - 4x_2 + 3x_3 - 2x_4 + 2x_5 = 48$$

13.
$$-x_2 - 2x_3 = 7$$
$$-x_1 + 2x_2 + 4x_3 = -20$$
$$x_1 - 2x_3 = 16$$
$$-x_2 + x_3 = -8$$
$$x_1 + x_2 + x_3 = 4$$

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14.

$$4x_1 + 3x_2 = 61$$
$$-3x_2 = -21$$
$$4x_1 - x_2 = 33$$
$$x_1 + 4x_2 = 38$$
$$-3x_1 + 4x_2 = -2$$

15.

$$-2x_1 - 2x_2 + 3x_4 - 4x_5 = 81$$
$$-4x_1 + x_2 + 3x_3 - 4x_4 = -26$$
$$-4x_1 - 3x_2 - 3x_3 + 2x_4 + x_5 = 51$$

16.

$$2x_1 - 2x_2 - 4x_3 = 32$$
$$3x_1 - 2x_2 + 3x_3 + 2x_4 = -49$$
$$-4x_1 + x_3 - 4x_4 = 56$$

17.

$$-4x_1 + x_2 - 4x_3 = -9$$
$$-2x_1 - 4x_2 - 4x_3 = 14$$
$$-x_2 + x_3 = 7$$
$$2x_2 - 2x_3 = -14$$
$$3x_2 - x_3 = -17$$

18.

$$3x_1 + 4x_2 - 4x_3 = 85$$
$$4x_2 + 2x_3 = 28$$
$$3x_1 + 3x_2 = 51$$
$$-2x_1 - 3x_2 = -44$$
$$-4x_1 - 3x_2 - x_3 = -52$$

19.

$$3x_1 - 3x_2 + 3x_3 + x_4 - 2x_5 - 4x_6 = 5$$
$$-2x_1 + x_2 + 3x_3 + 3x_4 + 2x_5 - x_6 = -8$$

20.

$$-4x_1 + 2x_2 - 3x_3 - 3x_4 = 57$$
$$4x_2 + 3x_3 = 22$$
$$4x_1 + 2x_3 + 2x_4 = -46$$
$$4x_1 - 4x_2 + 2x_3 + 2x_4 = -62$$

21.

$$2x_1 + x_2 - 3x_3 = 2$$
$$4x_1 + 3x_2 - 2x_3 = -11$$
$$-4x_3 = 12$$
$$-2x_1 + x_2 + 3x_3 = -8$$
$$2x_1 - 2x_2 + 3x_3 = -7$$

22.

$$x_1 - 3x_2 + 2x_3 + 4x_4 - 4x_5 + 4x_6 = 23$$
$$4x_1 - 4x_2 + 2x_3 - 3x_4 - x_5 - 2x_6 = 19$$

23.

$$-2x_1 + 2x_2 + 3x_3 - 4x_4 - 2x_5 = 19$$
$$3x_1 - 3x_2 - 3x_3 - x_4 + 3x_5 = -14$$
$$-2x_2 - 3x_4 - 2x_5 = 19$$

24.

$$-2x_2 + 3x_3 + x_4 = 16$$
$$3x_1 + x_2 - x_3 + 2x_4 = 26$$
$$4x_1 - 3x_2 + 3x_3 - 3x_4 = 15$$
$$2x_1 - 2x_3 - 2x_4 = -12$$

25.

$$4x_1 - 4x_2 = -64$$
$$-3x_1 + 4x_2 = 56$$
$$x_1 - x_2 = -16$$
$$-2x_1 + 4x_2 = 48$$
$$2x_1 + x_2 = -8$$

Solutions:

1.

$$-2x_1 - 2x_2 + 2x_3 = -6$$
$$4x_1 - x_2 + x_3 = 32$$
$$x_1 - x_2 - x_3 = 1$$
$$-3x_1 - 4x_2 + 3x_3 = -10$$
$$\left[\begin{array}{ccc|c} -2 & -2 & 2 & -6 \\ 4 & -1 & 1 & 32 \\ 1 & -1 & -1 & 1 \\ -3 & -4 & 3 & -10 \end{array}\right]$$
$$R_1 \leftarrow \frac{-1}{2} R_1$$
$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 4 & -1 & 1 & 32 \\ 1 & -1 & -1 & 1 \\ -3 & -4 & 3 & -10 \end{array}\right]$$
$$R_2 \leftarrow R_2 - 4R_1$$
$$R_3 \leftarrow R_3 - R_1$$
$$R_4 \leftarrow R_4 + 3R_1$$
$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -5 & 5 & 20 \\ 0 & -2 & 0 & -2 \\ 0 & -1 & 0 & -1 \end{array}\right]$$
$$R_2 \leftarrow \frac{-1}{5} R_2$$
$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & -1 & -4 \\ 0 & -2 & 0 & -2 \\ 0 & -1 & 0 & -1 \end{array}\right]$$
$$R_1 \leftarrow R_1 - R_2$$
$$R_3 \leftarrow R_3 + 2R_2$$
$$R_4 \leftarrow R_4 + R_2$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & -2 & -10 \\ 0 & 0 & -1 & -5 \end{array}\right]$$
$$R_3 \leftarrow \frac{-1}{2} R_3$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & -1 & -5 \end{array}\right]$$
$$R_2 \leftarrow R_2 + R_3$$
$$R_4 \leftarrow R_4 + R_3$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$
$$x_1 = 7$$
$$x_2 = 1$$
$$x_3 = 5$$

2.

$$x_1 - 3x_2 = 0$$
$$-2x_1 + 3x_2 = 3$$
$$-4x_2 = 4$$
$$2x_1 + 4x_2 = -10$$
$$4x_1 - x_2 = -11$$
$$\left[\begin{array}{cc|c} 1 & -3 & 0 \\ -2 & 3 & 3 \\ 0 & -4 & 4 \\ 2 & 4 & -10 \\ 4 & -1 & -11 \end{array}\right]$$
$$R_2 \leftarrow R_2 + 2R_1$$
$$R_4 \leftarrow R_4 - 2R_1$$
$$R_5 \leftarrow R_5 - 4R_1$$
$$\left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & -3 & 3 \\ 0 & -4 & 4 \\ 0 & 10 & -10 \\ 0 & 11 & -11 \end{array}\right]$$
$$R_2 \leftarrow \frac{-1}{3} R_2$$
$$\left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & 1 & -1 \\ 0 & -4 & 4 \\ 0 & 10 & -10 \\ 0 & 11 & -11 \end{array}\right]$$
$$R_1 \leftarrow R_1 + 3R_2$$
$$R_3 \leftarrow R_3 + 4R_2$$
$$R_4 \leftarrow R_4 - 10R_2$$
$$R_5 \leftarrow R_5 - 11R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$x_1 = -3$$

$$x_2 = -1$$

3.

$$-x_1 + 2x_2 + 3x_3 = -36$$

$$-x_1 - 4x_2 - 2x_3 - x_4 + 4x_5 = 2$$

$$2x_3 + 2x_4 = -28$$

$$\left[\begin{array}{ccccc|c} -1 & 2 & 3 & 0 & 0 & -36 \\ -1 & -4 & -2 & -1 & 4 & 2 \\ 0 & 0 & 2 & 2 & 0 & -28 \end{array}\right]$$

$$R_1 \leftarrow -R_1$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & -3 & 0 & 0 & 36 \\ -1 & -4 & -2 & -1 & 4 & 2 \\ 0 & 0 & 2 & 2 & 0 & -28 \end{array}\right]$$

$$R_2 \leftarrow R_2 + R_1$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & -3 & 0 & 0 & 36 \\ 0 & -6 & -5 & -1 & 4 & 38 \\ 0 & 0 & 2 & 2 & 0 & -28 \end{array}\right]$$

$$R_2 \leftarrow \frac{-1}{6} R_2$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & -3 & 0 & 0 & 36 \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & \frac{-2}{3} & \frac{-19}{3} \\ 0 & 0 & 2 & 2 & 0 & -28 \end{array}\right]$$

$$R_1 \leftarrow R_1 + 2R_2$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & \frac{-4}{3} & \frac{1}{3} & \frac{-4}{3} & \frac{70}{3} \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & \frac{-2}{3} & \frac{-19}{3} \\ 0 & 0 & 2 & 2 & 0 & -28 \end{array}\right]$$

$$R_3 \leftarrow \frac{1}{2} R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & \frac{-4}{3} & \frac{1}{3} & \frac{-4}{3} & \frac{70}{3} \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & \frac{-2}{3} & \frac{-19}{3} \\ 0 & 0 & 1 & 1 & 0 & -14 \end{array}\right]$$

$$R_1 \leftarrow R_1 + \frac{4}{3} R_3$$

$$R_2 \leftarrow R_2 - \frac{5}{6} R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & \frac{5}{3} & \frac{-4}{3} & \frac{14}{3} \\ 0 & 1 & 0 & \frac{-2}{3} & \frac{-2}{3} & \frac{16}{3} \\ 0 & 0 & 1 & 1 & 0 & -14 \end{array}\right]$$

$$x_1 + \frac{5}{3} x_4 - \frac{4}{3} x_5 = \frac{14}{3}$$

$$x_2 - \frac{2}{3} x_4 - \frac{2}{3} x_5 = \frac{16}{3}$$

$$x_3 + x_4 = -14$$

$$x_1 = \frac{14}{3} - \frac{5}{3} r + \frac{4}{3} s$$

$$x_2 = \frac{16}{3} + \frac{2}{3} r + \frac{2}{3} s$$

$$x_3 = -14 - r$$

$$x_4 = r$$

$$x_5 = s$$

$$r, s \in \mathbb{R}$$

4.

$$-x_1 + 2x_3 + 3x_4 - 2x_5 - 2x_6 = 19$$

$$-2x_1 - 2x_2 - 3x_3 - 4x_4 + 3x_5 = -18$$

$$-3x_1 + 2x_2 - x_4 - x_6 = 5$$

$$\left[\begin{array}{cccccc|c} -1 & 0 & 2 & 3 & -2 & -2 & 19 \\ -2 & -2 & -3 & -4 & 3 & 0 & -18 \\ -3 & 2 & 0 & -1 & 0 & -1 & 5 \end{array}\right]$$

$$R_1 \leftarrow -R_1$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -2 & -3 & 2 & 2 & -19 \\ -2 & -2 & -3 & -4 & 3 & 0 & -18 \\ -3 & 2 & 0 & -1 & 0 & -1 & 5 \end{array}\right]$$

$$R_2 \leftarrow R_2 + 2R_1$$

$$R_3 \leftarrow R_3 + 3R_1$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -2 & -3 & 2 & 2 & -19 \\ 0 & -2 & -7 & -10 & 7 & 4 & -56 \\ 0 & 2 & -6 & -10 & 6 & 5 & -52 \end{array}\right]$$

$$R_2 \leftarrow R_2 + \frac{3}{2} R_3$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -2 & -3 & 2 & 2 & -19 \\ 0 & 1 & -16 & -25 & 16 & \frac{23}{2} & -134 \\ 0 & 2 & -6 & -10 & 6 & 5 & -52 \end{array}\right]$$

$$R_3 \leftarrow R_3 - 2R_2$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -2 & -3 & 2 & 2 & -19 \\ 0 & 1 & -16 & -25 & 16 & \frac{23}{2} & -134 \\ 0 & 0 & 26 & 40 & -26 & -18 & 216 \end{array}\right]$$

$$R_3 \leftarrow \frac{1}{26}R_3$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -2 & -3 & 2 & 2 & -19 \\ 0 & 1 & -16 & -25 & 16 & \frac{23}{2} & -134 \\ 0 & 0 & 1 & \frac{20}{13} & -1 & \frac{-9}{13} & \frac{108}{13} \end{array}\right]$$

$$R_1 \leftarrow R_1 + 2R_3$$

$$R_2 \leftarrow R_2 + 16R_3$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & \frac{1}{13} & 0 & \frac{8}{13} & \frac{-31}{13} \\ 0 & 1 & 0 & \frac{-5}{13} & 0 & \frac{11}{26} & \frac{-14}{13} \\ 0 & 0 & 1 & \frac{20}{13} & -1 & \frac{-9}{13} & \frac{108}{13} \end{array}\right]$$

$$x_1 + \frac{1}{13}x_4 + \frac{8}{13}x_6 = \frac{-31}{13}$$

$$x_2 - \frac{5}{13}x_4 + \frac{11}{26}x_6 = \frac{-14}{13}$$

$$x_3 + \frac{20}{13}x_4 - x_5 - \frac{9}{13}x_6 = \frac{108}{13}$$

$$x_1 = \frac{-31}{13} - \frac{1}{13}r - \frac{8}{13}t$$

$$x_2 = \frac{-14}{13} + \frac{5}{13}r - \frac{11}{26}t$$

$$x_3 = \frac{108}{13} - \frac{20}{13}r + s + \frac{9}{13}t$$

$$x_4 = r$$

$$x_5 = s$$

$$x_6 = t$$

$$r, s, t \in \mathbb{R}$$

5.

$$2x_1 - 2x_2 + 2x_3 + 4x_4 = -48$$

$$-4x_1 - 4x_2 - 2x_3 - 2x_4 = -22$$

$$3x_1 + 2x_3 - 2x_4 = 17$$

$$-x_1 - x_3 + 3x_4 = -26$$

$$\left[\begin{array}{cccc|c} 2 & -2 & 2 & 4 & -48 \\ -4 & -4 & -2 & -2 & -22 \\ 3 & 0 & 2 & -2 & 17 \\ -1 & 0 & -1 & 3 & -26 \end{array}\right]$$

$$R_1 \leftarrow \frac{1}{2}R_1$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 2 & -24 \\ -4 & -4 & -2 & -2 & -22 \\ 3 & 0 & 2 & -2 & 17 \\ -1 & 0 & -1 & 3 & -26 \end{array}\right]$$

$$R_2 \leftarrow R_2 + 4R_1$$

$$R_3 \leftarrow R_3 - 3R_1$$

$$R_4 \leftarrow R_4 + R_1$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 2 & -24 \\ 0 & -8 & 2 & 6 & -118 \\ 0 & 3 & -1 & -8 & 89 \\ 0 & -1 & 0 & 5 & -50 \end{array}\right]$$

$$R_2 \leftarrow R_2 + 3R_3$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 2 & -24 \\ 0 & 1 & -1 & -18 & 149 \\ 0 & 3 & -1 & -8 & 89 \\ 0 & -1 & 0 & 5 & -50 \end{array}\right]$$

$$R_1 \leftarrow R_1 + R_2$$

$$R_3 \leftarrow R_3 - 3R_2$$

$$R_4 \leftarrow R_4 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -16 & 125 \\ 0 & 1 & -1 & -18 & 149 \\ 0 & 0 & 2 & 46 & -358 \\ 0 & 0 & -1 & -13 & 99 \end{array}\right]$$

$$R_3 \leftarrow \frac{1}{2}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -16 & 125 \\ 0 & 1 & -1 & -18 & 149 \\ 0 & 0 & 1 & 23 & -179 \\ 0 & 0 & -1 & -13 & 99 \end{array}\right]$$

$$R_2 \leftarrow R_2 + R_3$$

$$R_4 \leftarrow R_4 + R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -16 & 125 \\ 0 & 1 & 0 & 5 & -30 \\ 0 & 0 & 1 & 23 & -179 \\ 0 & 0 & 0 & 10 & -80 \end{array}\right]$$

$$R_4 \leftarrow \frac{1}{10}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -16 & 125 \\ 0 & 1 & 0 & 5 & -30 \\ 0 & 0 & 1 & 23 & -179 \\ 0 & 0 & 0 & 1 & -8 \end{array}\right]$$

$$R_1 \leftarrow R_1 + 16R_4$$

$$R_2 \leftarrow R_2 - 5R_4$$

$$R_3 \leftarrow R_3 - 23R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & -8 \end{array}\right]$$

$$x_1 = -3$$

$$x_2 = 10$$

$$x_3 = 5$$

$$x_4 = -8$$

6.

$$-3x_1 + 2x_2 + 2x_3 + 3x_4 = 3$$

$$x_1 + 2x_2 - 4x_4 = -12$$

$$2x_1 + 3x_2 - 3x_3 + 3x_4 = -77$$

$$\left[\begin{array}{cccc|c} -3 & 2 & 2 & 3 & 3 \\ 1 & 2 & 0 & -4 & -12 \\ 2 & 3 & -3 & 3 & -77 \end{array}\right]$$

$$R_1 \leftarrow R_1 + 4R_2$$

$$\left[\begin{array}{cccc|c} 1 & 10 & 2 & -13 & -45 \\ 1 & 2 & 0 & -4 & -12 \\ 2 & 3 & -3 & 3 & -77 \end{array}\right]$$

$$R_2 \leftarrow R_2 - R_1$$

$$R_3 \leftarrow R_3 - 2R_1$$

$$\left[\begin{array}{cccc|c} 1 & 10 & 2 & -13 & -45 \\ 0 & -8 & -2 & 9 & 33 \\ 0 & -17 & -7 & 29 & 13 \end{array}\right]$$

$$R_2 \leftarrow \frac{-1}{8}R_2$$

$$\left[\begin{array}{cccc|c} 1 & 10 & 2 & -13 & -45 \\ 0 & 1 & \frac{1}{4} & \frac{-9}{8} & \frac{-33}{8} \\ 0 & -17 & -7 & 29 & 13 \end{array}\right]$$

$$R_1 \leftarrow R_1 - 10R_2$$

$$R_3 \leftarrow R_3 + 17R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{-1}{2} & \frac{-7}{4} & \frac{-15}{4} \\ 0 & 1 & \frac{1}{4} & \frac{-9}{8} & \frac{-33}{8} \\ 0 & 0 & \frac{-11}{4} & \frac{79}{8} & \frac{-457}{8} \end{array}\right]$$

$$R_3 \leftarrow \frac{-4}{11}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{-1}{2} & \frac{-7}{4} & \frac{-15}{4} \\ 0 & 1 & \frac{1}{4} & \frac{-9}{8} & \frac{-33}{8} \\ 0 & 0 & 1 & \frac{-79}{22} & \frac{457}{22} \end{array}\right]$$

$$R_1 \leftarrow R_1 + \frac{1}{2}R_3$$

$$R_2 \leftarrow R_2 - \frac{1}{4}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{-39}{11} & \frac{73}{11} \\ 0 & 1 & 0 & \frac{-5}{22} & \frac{-205}{22} \\ 0 & 0 & 1 & \frac{-79}{22} & \frac{457}{22} \end{array}\right]$$

$$x_1 - \frac{39}{11}x_4 = \frac{73}{11}$$

$$x_2 - \frac{5}{22}x_4 = \frac{-205}{22}$$

$$x_3 - \frac{79}{22}x_4 = \frac{457}{22}$$

$$x_1 = \frac{73}{11} + \frac{39}{11}r$$

$$x_2 = \frac{-205}{22} + \frac{5}{22}r$$

$$x_3 = \frac{457}{22} + \frac{79}{22}r$$

$$x_4 = r$$

$$r \in \mathbb{R}$$

7.

$$-4x_1 - 3x_2 - x_3 + 2x_4 + 4x_5 - 2x_6 = 71$$

$$-3x_1 - 4x_3 + 3x_4 + 2x_5 + 4x_6 = 69$$

$$3x_1 + 2x_2 + x_3 + 3x_4 = 9$$

$$\left[\begin{array}{cccccc|c} -4 & -3 & -1 & 2 & 4 & -2 & 71 \\ -3 & 0 & -4 & 3 & 2 & 4 & 69 \\ 3 & 2 & 1 & 3 & 0 & 0 & 9 \end{array}\right]$$

$$R_1 \leftarrow R_1 + \frac{5}{3}R_3$$

$$\left[\begin{array}{cccccc|c} 1 & \frac{1}{3} & \frac{2}{3} & 7 & 4 & -2 & 86 \\ -3 & 0 & -4 & 3 & 2 & 4 & 69 \\ 3 & 2 & 1 & 3 & 0 & 0 & 9 \end{array}\right]$$

$$R_2 \leftarrow R_2 + 3R_1$$

$$R_3 \leftarrow R_3 - 3R_1$$

$$\left[\begin{array}{cccccc|c} 1 & \frac{1}{3} & \frac{2}{3} & 7 & 4 & -2 & 86 \\ 0 & 1 & -2 & 24 & 14 & -2 & 327 \\ 0 & 1 & -1 & -18 & -12 & 6 & -249 \end{array}\right]$$

$$R_2 \leftarrow 1R_2$$

$$\left[\begin{array}{cccccc|c} 1 & \frac{1}{3} & \frac{2}{3} & 7 & 4 & -2 & 86 \\ 0 & 1 & -2 & 24 & 14 & -2 & 327 \\ 0 & 1 & -1 & -18 & -12 & 6 & -249 \end{array}\right]$$

$$R_1 \leftarrow R_1 - \frac{1}{3}R_2$$

$$R_3 \leftarrow R_3 - 1R_2$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & \frac{4}{3} & -1 & \frac{-2}{3} & \frac{-4}{3} & -23 \\ 0 & 1 & -2 & 24 & 14 & -2 & 327 \\ 0 & 0 & 1 & -42 & -26 & 8 & -576 \end{array}\right]$$

$$R_3 \leftarrow 1R_3$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & \frac{4}{3} & -1 & \frac{-2}{3} & \frac{-4}{3} & -23 \\ 0 & 1 & -2 & 24 & 14 & -2 & 327 \\ 0 & 0 & 1 & -42 & -26 & 8 & -576 \end{array}\right]$$

$$R_1 \leftarrow R_1 - \frac{4}{3}R_3$$

$$R_2 \leftarrow R_2 + 2R_3$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 55 & 34 & -12 & 745 \\ 0 & 1 & 0 & -60 & -38 & 14 & -825 \\ 0 & 0 & 1 & -42 & -26 & 8 & -576 \end{array}\right]$$

$$x_1 + 55x_4 + 34x_5 - 12x_6 = 745$$

$$x_2 - 60x_4 - 38x_5 + 14x_6 = -825$$

$$x_3 - 42x_4 - 26x_5 + 8x_6 = -576$$

$$x_1 = 745 - 55r - 34s + 12t$$

$$x_2 = -825 + 60r + 38s - 14t$$

$$x_3 = -576 + 42r + 26s - 8t$$

$$x_4 = r$$

$$x_5 = s$$

$$x_6 = t$$

$$r, s, t \in \mathbb{R}$$

8.

$$-4x_1 - 4x_2 - x_3 - 2x_4 + 3x_5 - 3x_6 = 18$$

$$-4x_1 - x_2 + x_3 + 4x_4 - 2x_5 + x_6 = 0$$

$$\left[\begin{array}{cccccc|c} -4 & -4 & -1 & -2 & 3 & -3 & 18 \\ -4 & -1 & 1 & 4 & -2 & 1 & 0 \end{array}\right]$$

$$R_1 \leftarrow R_1 - \frac{5}{4}R_2$$

$$\left[\begin{array}{cccccc|c} 1 & \frac{-11}{4} & \frac{-9}{4} & -7 & \frac{11}{2} & \frac{-17}{4} & 18 \\ -4 & -1 & 1 & 4 & -2 & 1 & 0 \end{array}\right]$$

$$R_2 \leftarrow R_2 + 4R_1$$

$$\left[\begin{array}{cccccc|c} 1 & \frac{-11}{4} & \frac{-9}{4} & -7 & \frac{11}{2} & \frac{-17}{4} & 18 \\ 0 & -12 & -8 & -24 & 20 & -16 & 72 \end{array}\right]$$

$$R_2 \leftarrow \frac{-1}{12}R_2$$

$$\left[\begin{array}{cccccc|c} 1 & \frac{-11}{4} & \frac{-9}{4} & -7 & \frac{11}{2} & \frac{-17}{4} & 18 \\ 0 & 1 & \frac{2}{3} & 2 & \frac{-5}{3} & \frac{4}{3} & -6 \end{array}\right]$$

$$R_1 \leftarrow R_1 + \frac{11}{4}R_2$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & \frac{-5}{12} & \frac{-3}{2} & \frac{11}{12} & \frac{-7}{12} & \frac{3}{2} \\ 0 & 1 & \frac{2}{3} & 2 & \frac{-5}{3} & \frac{4}{3} & -6 \end{array}\right]$$

$$x_1 - \frac{5}{12}x_3 - \frac{3}{2}x_4 + \frac{11}{12}x_5 - \frac{7}{12}x_6 = \frac{3}{2}$$

$$x_2 + \frac{2}{3}x_3 + 2x_4 - \frac{5}{3}x_5 + \frac{4}{3}x_6 = -6$$

$$x_1 = \frac{3}{2} + \frac{5}{12}r + \frac{3}{2}s - \frac{11}{12}t + \frac{7}{12}k$$

$$x_2 = -6 - \frac{2}{3}r - 2s + \frac{5}{3}t - \frac{4}{3}k$$

$$x_3 = r$$

$$x_4 = s$$

$$x_5 = t$$

$$x_6 = k$$

$$r, s, t, k \in \mathbb{R}$$

9.

$$-x_1 - 3x_2 - 2x_3 - 2x_4 = 36$$

$$2x_1 + 4x_2 - 3x_3 + 3x_4 = -7$$

$$-x_1 - 2x_2 - 4x_3 + 2x_4 = 30$$

$$\left[\begin{array}{cccc|c} -1 & -3 & -2 & -2 & 36 \\ 2 & 4 & -3 & 3 & -7 \\ -1 & -2 & -4 & 2 & 30 \end{array}\right]$$

$$R_1 \leftarrow -R_1$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 2 & -36 \\ 2 & 4 & -3 & 3 & -7 \\ -1 & -2 & -4 & 2 & 30 \end{array}\right]$$

$$R_2 \leftarrow R_2 - 2R_1$$

$$R_3 \leftarrow R_3 + R_1$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 2 & -36 \\ 0 & -2 & -7 & -1 & 65 \\ 0 & 1 & -2 & 4 & -6 \end{array}\right]$$

$$R_2 \leftarrow R_2 + 3R_3$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 2 & -36 \\ 0 & 1 & -13 & 11 & 47 \\ 0 & 1 & -2 & 4 & -6 \end{array}\right]$$

$$R_1 \leftarrow R_1 - 3R_2$$

$$R_3 \leftarrow R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 41 & -31 & -177 \\ 0 & 1 & -13 & 11 & 47 \\ 0 & 0 & 11 & -7 & -53 \end{array}\right]$$

$$R_3 \leftarrow \frac{1}{11}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 41 & -31 & -177 \\ 0 & 1 & -13 & 11 & 47 \\ 0 & 0 & 1 & \frac{-7}{11} & \frac{-53}{11} \end{array}\right]$$

$$R_1 \leftarrow R_1 - 41R_3$$

$$R_2 \leftarrow R_2 + 13R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{-54}{11} & \frac{226}{11} \\ 0 & 1 & 0 & \frac{30}{11} & \frac{-172}{11} \\ 0 & 0 & 1 & \frac{-7}{11} & \frac{-53}{11} \end{array}\right]$$

$$x_1 - \frac{54}{11}x_4 = \frac{226}{11}$$

$$x_2 + \frac{30}{11}x_4 = \frac{-172}{11}$$

$$x_3 - \frac{7}{11}x_4 = \frac{-53}{11}$$

$$x_1 = \frac{226}{11} + \frac{54}{11}r$$

$$x_2 = \frac{-172}{11} - \frac{30}{11}r$$

$$x_3 = \frac{-53}{11} + \frac{7}{11}r$$

$$x_4 = r$$

$$r \in \mathbb{R}$$

10.

$$x_1 + 3x_2 - 2x_3 = 38$$

$$3x_1 - 3x_2 - 2x_3 = -38$$

$$-x_1 - x_2 - 4x_3 = 30$$

$$-3x_1 + 3x_2 = 54$$

$$-4x_1 + 3x_3 = 8$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 38 \\ 3 & -3 & -2 & -38 \\ -1 & -1 & -4 & 30 \\ -3 & 3 & 0 & 54 \\ -4 & 0 & 3 & 8 \end{array}\right]$$

$$R_2 \leftarrow R_2 - 3R_1$$

$$R_3 \leftarrow R_3 + R_1$$

$$R_4 \leftarrow R_4 + 3R_1$$

$$R_5 \leftarrow R_5 + 4R_1$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 38 \\ 0 & -12 & 4 & -152 \\ 0 & 2 & -6 & 68 \\ 0 & 12 & -6 & 168 \\ 0 & 12 & -5 & 160 \end{array}\right]$$

$$R_2 \leftarrow R_2 + \frac{13}{2}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 38 \\ 0 & 1 & -35 & 290 \\ 0 & 2 & -6 & 68 \\ 0 & 12 & -6 & 168 \\ 0 & 12 & -5 & 160 \end{array}\right]$$

$$R_1 \leftarrow R_1 - 3R_2$$

$$R_3 \leftarrow R_3 - 2R_2$$

$$R_4 \leftarrow R_4 - 12R_2$$

$$R_5 \leftarrow R_5 - 12R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 103 & -832 \\ 0 & 1 & -35 & 290 \\ 0 & 0 & 64 & -512 \\ 0 & 0 & 414 & -3312 \\ 0 & 0 & 415 & -3320 \end{array}\right]$$

$$R_3 \leftarrow \frac{1}{64}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 103 & -832 \\ 0 & 1 & -35 & 290 \\ 0 & 0 & 1 & -8 \\ 0 & 0 & 414 & -3312 \\ 0 & 0 & 415 & -3320 \end{array}\right]$$

$$R_1 \leftarrow R_1 - 103R_3$$

$$R_2 \leftarrow R_2 + 35R_3$$

$$R_4 \leftarrow R_4 - 414R_3$$

$$R_5 \leftarrow R_5 - 415R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 = -8$$

$$x_2 = 10$$

$$x_3 = -8$$

11.

$$2x_2 - x_3 = 8$$
$$3x_1 - 3x_2 + x_3 = -25$$
$$-x_1 + 4x_2 + 4x_3 = -3$$
$$-x_2 - 3x_3 = 10$$
$$\left[\begin{array}{ccc|c} 0 & 2 & -1 & 8 \\ 3 & -3 & 1 & -25 \\ -1 & 4 & 4 & -3 \\ 0 & -1 & -3 & 10 \end{array}\right]$$
$$R_1 \leftrightarrow R_2$$
$$\left[\begin{array}{ccc|c} 3 & -3 & 1 & -25 \\ 0 & 2 & -1 & 8 \\ -1 & 4 & 4 & -3 \\ 0 & -1 & -3 & 10 \end{array}\right]$$
$$R_1 \leftarrow R_1 + 2R_3$$
$$\left[\begin{array}{ccc|c} 1 & 5 & 9 & -31 \\ 0 & 2 & -1 & 8 \\ -1 & 4 & 4 & -3 \\ 0 & -1 & -3 & 10 \end{array}\right]$$
$$R_3 \leftarrow R_3 + R_1$$
$$\left[\begin{array}{ccc|c} 1 & 5 & 9 & -31 \\ 0 & 2 & -1 & 8 \\ 0 & 9 & 13 & -34 \\ 0 & -1 & -3 & 10 \end{array}\right]$$
$$R_2 \leftarrow R_2 + R_4$$
$$\left[\begin{array}{ccc|c} 1 & 5 & 9 & -31 \\ 0 & 1 & -4 & 18 \\ 0 & 9 & 13 & -34 \\ 0 & -1 & -3 & 10 \end{array}\right]$$
$$R_1 \leftarrow R_1 - 5R_2$$
$$R_3 \leftarrow R_3 - 9R_2$$
$$R_4 \leftarrow R_4 + R_2$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 29 & -121 \\ 0 & 1 & -4 & 18 \\ 0 & 0 & 49 & -196 \\ 0 & 0 & -7 & 28 \end{array}\right]$$
$$R_3 \leftarrow R_3 + \frac{48}{7}R_4$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 29 & -121 \\ 0 & 1 & -4 & 18 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & -7 & 28 \end{array}\right]$$
$$R_1 \leftarrow R_1 - 29R_3$$
$$R_2 \leftarrow R_2 + 4R_3$$
$$R_4 \leftarrow R_4 + 7R_3$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$
$$x_1 = -5$$
$$x_2 = 2$$
$$x_3 = -4$$

12.

$$-3x_1 + 4x_2 - 4x_4 - 2x_5 = 21$$
$$2x_1 - 4x_2 + 4x_3 + 4x_4 + 3x_5 = 5$$
$$3x_1 - 4x_2 + 3x_3 - 2x_4 + 2x_5 = 48$$
$$\left[\begin{array}{ccccc|c} -3 & 4 & 0 & -4 & -2 & 21 \\ 2 & -4 & 4 & 4 & 3 & 5 \\ 3 & -4 & 3 & -2 & 2 & 48 \end{array}\right]$$
$$R_1 \leftarrow R_1 + 2R_2$$
$$\left[\begin{array}{ccccc|c} 1 & -4 & 8 & 4 & 4 & 31 \\ 2 & -4 & 4 & 4 & 3 & 5 \\ 3 & -4 & 3 & -2 & 2 & 48 \end{array}\right]$$
$$R_2 \leftarrow R_2 - 2R_1$$
$$R_3 \leftarrow R_3 - 3R_1$$
$$\left[\begin{array}{ccccc|c} 1 & -4 & 8 & 4 & 4 & 31 \\ 0 & 4 & -12 & -4 & -5 & -57 \\ 0 & 8 & -21 & -14 & -10 & -45 \end{array}\right]$$
$$R_2 \leftarrow \frac{1}{4}R_2$$
$$\left[\begin{array}{ccccc|c} 1 & -4 & 8 & 4 & 4 & 31 \\ 0 & 1 & -3 & -1 & \frac{-5}{4} & \frac{-57}{4} \\ 0 & 8 & -21 & -14 & -10 & -45 \end{array}\right]$$
$$R_1 \leftarrow R_1 + 4R_2$$
$$R_3 \leftarrow R_3 - 8R_2$$
$$\left[\begin{array}{ccccc|c} 1 & 0 & -4 & 0 & -1 & -26 \\ 0 & 1 & -3 & -1 & \frac{-5}{4} & \frac{-57}{4} \\ 0 & 0 & 3 & -6 & 0 & 69 \end{array}\right]$$
$$R_3 \leftarrow \frac{1}{3}R_3$$
$$\left[\begin{array}{ccccc|c} 1 & 0 & -4 & 0 & -1 & -26 \\ 0 & 1 & -3 & -1 & \frac{-5}{4} & \frac{-57}{4} \\ 0 & 0 & 1 & -2 & 0 & 23 \end{array}\right]$$
$$R_1 \leftarrow R_1 + 4R_3$$
$$R_2 \leftarrow R_2 + 3R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -8 & -1 & 66 \\ 0 & 1 & 0 & -7 & \frac{-5}{4} & \frac{219}{4} \\ 0 & 0 & 1 & -2 & 0 & 23 \end{array}\right]$$

$$x_1 - 8x_4 - x_5 = 66$$

$$x_2 - 7x_4 - \frac{5}{4}x_5 = \frac{219}{4}$$

$$x_3 - 2x_4 = 23$$

$$x_1 = 66 + 8r + s$$

$$x_2 = \frac{219}{4} + 7r + \frac{5}{4}s$$

$$x_3 = 23 + 2r$$

$$x_4 = r$$

$$x_5 = s$$

$$r, s \in \mathbb{R}$$

13.

$$-x_2 - 2x_3 = 7$$

$$-x_1 + 2x_2 + 4x_3 = -20$$

$$x_1 - 2x_3 = 16$$

$$-x_2 + x_3 = -8$$

$$x_1 + x_2 + x_3 = 4$$

$$\left[\begin{array}{ccc|c} 0 & -1 & -2 & 7 \\ -1 & 2 & 4 & -20 \\ 1 & 0 & -2 & 16 \\ 0 & -1 & 1 & -8 \\ 1 & 1 & 1 & 4 \end{array}\right]$$

$$R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} -1 & 2 & 4 & -20 \\ 0 & -1 & -2 & 7 \\ 1 & 0 & -2 & 16 \\ 0 & -1 & 1 & -8 \\ 1 & 1 & 1 & 4 \end{array}\right]$$

$$R_1 \leftarrow R_1 + 2R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 12 \\ 0 & -1 & -2 & 7 \\ 1 & 0 & -2 & 16 \\ 0 & -1 & 1 & -8 \\ 1 & 1 & 1 & 4 \end{array}\right]$$

$$R_3 \leftarrow R_3 - R_1$$

$$R_5 \leftarrow R_5 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 12 \\ 0 & -1 & -2 & 7 \\ 0 & -2 & -2 & 4 \\ 0 & -1 & 1 & -8 \\ 0 & -1 & 1 & -8 \end{array}\right]$$

$$R_2 \leftarrow R_2 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 12 \\ 0 & 1 & 0 & 3 \\ 0 & -2 & -2 & 4 \\ 0 & -1 & 1 & -8 \\ 0 & -1 & 1 & -8 \end{array}\right]$$

$$R_1 \leftarrow R_1 - 2R_2$$

$$R_3 \leftarrow R_3 + 2R_2$$

$$R_4 \leftarrow R_4 + R_2$$

$$R_5 \leftarrow R_5 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -2 & 10 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

$$R_3 \leftarrow \frac{-1}{2}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

$$R_4 \leftarrow R_4 - R_3$$

$$R_5 \leftarrow R_5 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 = 6$$

$$x_2 = 3$$

$$x_3 = -5$$

14.

$$4x_1 + 3x_2 = 61$$
$$-3x_2 = -21$$
$$4x_1 - x_2 = 33$$
$$x_1 + 4x_2 = 38$$
$$-3x_1 + 4x_2 = -2$$

$$\left[\begin{array}{cc|c} 4 & 3 & 61 \\ 0 & -3 & -21 \\ 4 & -1 & 33 \\ 1 & 4 & 38 \\ -3 & 4 & -2 \end{array}\right]$$

$$R_1 \leftarrow R_1 - 3R_4$$

$$\left[\begin{array}{cc|c} 1 & -9 & -53 \\ 0 & -3 & -21 \\ 4 & -1 & 33 \\ 1 & 4 & 38 \\ -3 & 4 & -2 \end{array}\right]$$

$$R_3 \leftarrow R_3 - 4R_1$$
$$R_4 \leftarrow R_4 - R_1$$
$$R_5 \leftarrow R_5 + 3R_1$$

$$\left[\begin{array}{cc|c} 1 & -9 & -53 \\ 0 & -3 & -21 \\ 0 & 35 & 245 \\ 0 & 13 & 91 \\ 0 & -23 & -161 \end{array}\right]$$

$$R_2 \leftarrow \frac{-1}{3}R_2$$

$$\left[\begin{array}{cc|c} 1 & -9 & -53 \\ 0 & 1 & 7 \\ 0 & 35 & 245 \\ 0 & 13 & 91 \\ 0 & -23 & -161 \end{array}\right]$$

$$R_1 \leftarrow R_1 + 9R_2$$
$$R_3 \leftarrow R_3 - 35R_2$$
$$R_4 \leftarrow R_4 - 13R_2$$
$$R_5 \leftarrow R_5 + 23R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & 10 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$x_1 = 10$$
$$x_2 = 7$$

15.

$$-2x_1 - 2x_2 + 3x_4 - 4x_5 = 81$$
$$-4x_1 + x_2 + 3x_3 - 4x_4 = -26$$
$$-4x_1 - 3x_2 - 3x_3 + 2x_4 + x_5 = 51$$

$$\left[\begin{array}{ccccc|c} -2 & -2 & 0 & 3 & -4 & 81 \\ -4 & 1 & 3 & -4 & 0 & -26 \\ -4 & -3 & -3 & 2 & 1 & 51 \end{array}\right]$$

$$R_1 \leftarrow \frac{-1}{2}R_1$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & \frac{-3}{2} & 2 & \frac{-81}{2} \\ -4 & 1 & 3 & -4 & 0 & -26 \\ -4 & -3 & -3 & 2 & 1 & 51 \end{array}\right]$$

$$R_2 \leftarrow R_2 + 4R_1$$
$$R_3 \leftarrow R_3 + 4R_1$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & \frac{-3}{2} & 2 & \frac{-81}{2} \\ 0 & 5 & 3 & -10 & 8 & -188 \\ 0 & 1 & -3 & -4 & 9 & -111 \end{array}\right]$$

$$R_2 \leftarrow R_2 - 4R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & \frac{-3}{2} & 2 & \frac{-81}{2} \\ 0 & 1 & 15 & 6 & -28 & 256 \\ 0 & 1 & -3 & -4 & 9 & -111 \end{array}\right]$$

$$R_1 \leftarrow R_1 - R_2$$
$$R_3 \leftarrow R_3 - R_2$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -15 & \frac{-15}{2} & 30 & \frac{-593}{2} \\ 0 & 1 & 15 & 6 & -28 & 256 \\ 0 & 0 & -18 & -10 & 37 & -367 \end{array}\right]$$

$$R_3 \leftarrow \frac{-1}{18}R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -15 & \frac{-15}{2} & 30 & \frac{-593}{2} \\ 0 & 1 & 15 & 6 & -28 & 256 \\ 0 & 0 & 1 & \frac{5}{9} & \frac{-37}{18} & \frac{367}{18} \end{array}\right]$$

$$R_1 \leftarrow R_1 + 15R_3$$
$$R_2 \leftarrow R_2 - 15R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & \frac{5}{6} & \frac{-5}{6} & \frac{28}{3} \\ 0 & 1 & 0 & \frac{-7}{3} & \frac{17}{6} & \frac{-299}{6} \\ 0 & 0 & 1 & \frac{5}{9} & \frac{-37}{18} & \frac{367}{18} \end{array}\right]$$

$$x_1 + \frac{5}{6}x_4 - \frac{5}{6}x_5 = \frac{28}{3}$$
$$x_2 - \frac{7}{3}x_4 + \frac{17}{6}x_5 = \frac{-299}{6}$$
$$x_3 + \frac{5}{9}x_4 - \frac{37}{18}x_5 = \frac{367}{18}$$

$$x_1 = \frac{28}{3} - \frac{5}{6}r + \frac{5}{6}s$$
$$x_2 = \frac{-299}{6} + \frac{7}{3}r - \frac{17}{6}s$$
$$x_3 = \frac{367}{18} - \frac{5}{9}r + \frac{37}{18}s$$
$$x_4 = r$$
$$x_5 = s$$
$$r, s \in \mathbb{R}$$

16.

$$2x_1 - 2x_2 - 4x_3 = 32$$
$$3x_1 - 2x_2 + 3x_3 + 2x_4 = -49$$
$$-4x_1 + x_3 - 4x_4 = 56$$
$$\left[\begin{array}{cccc|c} 2 & -2 & -4 & 0 & 32 \\ 3 & -2 & 3 & 2 & -49 \\ -4 & 0 & 1 & -4 & 56 \end{array}\right]$$
$$R_1 \leftarrow \frac{1}{2}R_1$$
$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & 0 & 16 \\ 3 & -2 & 3 & 2 & -49 \\ -4 & 0 & 1 & -4 & 56 \end{array}\right]$$
$$R_2 \leftarrow R_2 - 3R_1$$
$$R_3 \leftarrow R_3 + 4R_1$$
$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & 0 & 16 \\ 0 & 1 & 9 & 2 & -97 \\ 0 & -4 & -7 & -4 & 120 \end{array}\right]$$
$$R_1 \leftarrow R_1 + R_2$$
$$R_3 \leftarrow R_3 + 4R_2$$
$$\left[\begin{array}{cccc|c} 1 & 0 & 7 & 2 & -81 \\ 0 & 1 & 9 & 2 & -97 \\ 0 & 0 & 29 & 4 & -268 \end{array}\right]$$
$$R_3 \leftarrow \frac{1}{29}R_3$$
$$\left[\begin{array}{cccc|c} 1 & 0 & 7 & 2 & -81 \\ 0 & 1 & 9 & 2 & -97 \\ 0 & 0 & 1 & \frac{4}{29} & \frac{-268}{29} \end{array}\right]$$
$$R_1 \leftarrow R_1 - 7R_3$$
$$R_2 \leftarrow R_2 - 9R_3$$
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{30}{29} & \frac{-473}{29} \\ 0 & 1 & 0 & \frac{22}{29} & \frac{-401}{29} \\ 0 & 0 & 1 & \frac{4}{29} & \frac{-268}{29} \end{array}\right]$$
$$x_1 + \frac{30}{29}x_4 = \frac{-473}{29}$$
$$x_2 + \frac{22}{29}x_4 = \frac{-401}{29}$$
$$x_3 + \frac{4}{29}x_4 = \frac{-268}{29}$$

$$x_1 = \frac{-473}{29} - \frac{30}{29}r$$
$$x_2 = \frac{-401}{29} - \frac{22}{29}r$$
$$x_3 = \frac{-268}{29} - \frac{4}{29}r$$
$$x_4 = r$$
$$r \in \mathbb{R}$$

17.

$$-4x_1 + x_2 - 4x_3 = -9$$
$$-2x_1 - 4x_2 - 4x_3 = 14$$
$$-x_2 + x_3 = 7$$
$$2x_2 - 2x_3 = -14$$
$$3x_2 - x_3 = -17$$
$$\left[\begin{array}{ccc|c} -4 & 1 & -4 & -9 \\ -2 & -4 & -4 & 14 \\ 0 & -1 & 1 & 7 \\ 0 & 2 & -2 & -14 \\ 0 & 3 & -1 & -17 \end{array}\right]$$
$$R_1 \leftarrow R_1 - \frac{5}{2}R_2$$
$$\left[\begin{array}{ccc|c} 1 & 11 & 6 & -44 \\ -2 & -4 & -4 & 14 \\ 0 & -1 & 1 & 7 \\ 0 & 2 & -2 & -14 \\ 0 & 3 & -1 & -17 \end{array}\right]$$
$$R_2 \leftarrow R_2 + 2R_1$$
$$\left[\begin{array}{ccc|c} 1 & 11 & 6 & -44 \\ 0 & 18 & 8 & -74 \\ 0 & -1 & 1 & 7 \\ 0 & 2 & -2 & -14 \\ 0 & 3 & -1 & -17 \end{array}\right]$$
$$R_2 \leftarrow R_2 + 17R_3$$
$$\left[\begin{array}{ccc|c} 1 & 11 & 6 & -44 \\ 0 & 1 & 25 & 45 \\ 0 & -1 & 1 & 7 \\ 0 & 2 & -2 & -14 \\ 0 & 3 & -1 & -17 \end{array}\right]$$
$$R_1 \leftarrow R_1 - 11R_2$$

$$R_3 \leftarrow R_3 + R_2$$

$$R_4 \leftarrow R_4 - 2R_2$$

$$R_5 \leftarrow R_5 - 3R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -269 & -539 \\ 0 & 1 & 25 & 45 \\ 0 & 0 & 26 & 52 \\ 0 & 0 & -52 & -104 \\ 0 & 0 & -76 & -152 \end{array}\right]$$

$$R_3 \leftarrow \frac{1}{26}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -269 & -539 \\ 0 & 1 & 25 & 45 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -52 & -104 \\ 0 & 0 & -76 & -152 \end{array}\right]$$

$$R_1 \leftarrow R_1 + 269R_3$$

$$R_2 \leftarrow R_2 - 25R_3$$

$$R_4 \leftarrow R_4 + 52R_3$$

$$R_5 \leftarrow R_5 + 76R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 = -1$$

$$x_2 = -5$$

$$x_3 = 2$$

18.

$$3x_1 + 4x_2 - 4x_3 = 85$$

$$4x_2 + 2x_3 = 28$$

$$3x_1 + 3x_2 = 51$$

$$-2x_1 - 3x_2 = -44$$

$$-4x_1 - 3x_2 - x_3 = -52$$

$$\left[\begin{array}{ccc|c} 3 & 4 & -4 & 85 \\ 0 & 4 & 2 & 28 \\ 3 & 3 & 0 & 51 \\ -2 & -3 & 0 & -44 \\ -4 & -3 & -1 & -52 \end{array}\right]$$

$$R_1 \leftarrow R_1 - \frac{2}{3}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 51 \\ 0 & 4 & 2 & 28 \\ 3 & 3 & 0 & 51 \\ -2 & -3 & 0 & -44 \\ -4 & -3 & -1 & -52 \end{array}\right]$$

$$R_3 \leftarrow R_3 - 3R_1$$

$$R_4 \leftarrow R_4 + 2R_1$$

$$R_5 \leftarrow R_5 + 4R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 51 \\ 0 & 4 & 2 & 28 \\ 0 & -3 & 12 & -102 \\ 0 & 1 & -8 & 58 \\ 0 & 5 & -17 & 152 \end{array}\right]$$

$$R_2 \leftarrow R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 51 \\ 0 & 1 & 14 & -74 \\ 0 & -3 & 12 & -102 \\ 0 & 1 & -8 & 58 \\ 0 & 5 & -17 & 152 \end{array}\right]$$

$$R_1 \leftarrow R_1 - 2R_2$$

$$R_3 \leftarrow R_3 + 3R_2$$

$$R_4 \leftarrow R_4 - R_2$$

$$R_5 \leftarrow R_5 - 5R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -32 & 199 \\ 0 & 1 & 14 & -74 \\ 0 & 0 & 54 & -324 \\ 0 & 0 & -22 & 132 \\ 0 & 0 & -87 & 522 \end{array}\right]$$

$$R_3 \leftarrow \frac{1}{54}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -32 & 199 \\ 0 & 1 & 14 & -74 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & -22 & 132 \\ 0 & 0 & -87 & 522 \end{array}\right]$$

$$R_1 \leftarrow R_1 + 32R_3$$

$$R_2 \leftarrow R_2 - 14R_3$$

$$R_4 \leftarrow R_4 + 22R_3$$

$$R_5 \leftarrow R_5 + 87R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 = 7$$

$$x_2 = 10$$

$$x_3 = -6$$

19.

$$3x_1 - 3x_2 + 3x_3 + x_4 - 2x_5 - 4x_6 = 5$$

$$-2x_1 + x_2 + 3x_3 + 3x_4 + 2x_5 - x_6 = -8$$

$$\left[\begin{array}{cccccc|c} 3 & -3 & 3 & 1 & -2 & -4 & 5 \\ -2 & 1 & 3 & 3 & 2 & -1 & -8 \end{array}\right]$$

$$R_1 \leftarrow R_1 + R_2$$

$$\left[\begin{array}{cccccc|c} 1 & -2 & 6 & 4 & 0 & -5 & -3 \\ -2 & 1 & 3 & 3 & 2 & -1 & -8 \end{array}\right]$$

$$R_2 \leftarrow R_2 + 2R_1$$

$$\left[\begin{array}{cccccc|c} 1 & -2 & 6 & 4 & 0 & -5 & -3 \\ 0 & -3 & 15 & 11 & 2 & -11 & -14 \end{array}\right]$$

$$R_2 \leftarrow \frac{-1}{3}R_2$$

$$\left[\begin{array}{cccccc|c} 1 & -2 & 6 & 4 & 0 & -5 & -3 \\ 0 & 1 & -5 & \frac{-11}{3} & \frac{-2}{3} & \frac{11}{3} & \frac{14}{3} \end{array}\right]$$

$$R_1 \leftarrow R_1 + 2R_2$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -4 & \frac{-10}{3} & \frac{-4}{3} & \frac{7}{3} & \frac{19}{3} \\ 0 & 1 & -5 & \frac{-11}{3} & \frac{-2}{3} & \frac{11}{3} & \frac{14}{3} \end{array}\right]$$

$$x_1 - 4x_3 - \frac{10}{3}x_4 - \frac{4}{3}x_5 + \frac{7}{3}x_6 = \frac{19}{3}$$

$$x_2 - 5x_3 - \frac{11}{3}x_4 - \frac{2}{3}x_5 + \frac{11}{3}x_6 = \frac{14}{3}$$

$$x_1 = \frac{19}{3} + 4r + \frac{10}{3}s + \frac{4}{3}t - \frac{7}{3}k$$

$$x_2 = \frac{14}{3} + 5r + \frac{11}{3}s + \frac{2}{3}t - \frac{11}{3}k$$

$$x_3 = r$$

$$x_4 = s$$

$$x_5 = t$$

$$x_6 = k$$

$$r, s, t, k \in \mathbb{R}$$

20.

$$-4x_1 + 2x_2 - 3x_3 - 3x_4 = 57$$

$$4x_2 + 3x_3 = 22$$

$$4x_1 + 2x_3 + 2x_4 = -46$$

$$4x_1 - 4x_2 + 2x_3 + 2x_4 = -62$$

$$\left[\begin{array}{cccc|c} -4 & 2 & -3 & -3 & 57 \\ 0 & 4 & 3 & 0 & 22 \\ 4 & 0 & 2 & 2 & -46 \\ 4 & -4 & 2 & 2 & -62 \end{array}\right]$$

$$R_1 \leftarrow R_1 + \frac{5}{4}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 2 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ 0 & 4 & 3 & 0 & 22 \\ 4 & 0 & 2 & 2 & -46 \\ 4 & -4 & 2 & 2 & -62 \end{array}\right]$$

$$R_3 \leftarrow R_3 - 4R_1$$

$$R_4 \leftarrow R_4 - 4R_1$$

$$\left[\begin{array}{cccc|c} 1 & 2 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ 0 & 4 & 3 & 0 & 22 \\ 0 & -8 & 4 & 4 & -44 \\ 0 & -12 & 4 & 4 & -60 \end{array}\right]$$

$$R_2 \leftarrow R_2 + \frac{1}{4}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 2 & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ 0 & 1 & 4 & 1 & 7 \\ 0 & -8 & 4 & 4 & -44 \\ 0 & -12 & 4 & 4 & -60 \end{array}\right]$$

$$R_1 \leftarrow R_1 - 2R_2$$

$$R_3 \leftarrow R_3 + 8R_2$$

$$R_4 \leftarrow R_4 + 12R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{-17}{2} & \frac{-5}{2} & \frac{-29}{2} \\ 0 & 1 & 4 & 1 & 7 \\ 0 & 0 & 36 & 12 & 12 \\ 0 & 0 & 52 & 16 & 24 \end{array}\right]$$

$$R_3 \leftarrow \frac{1}{36}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{-17}{2} & \frac{-5}{2} & \frac{-29}{2} \\ 0 & 1 & 4 & 1 & 7 \\ 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 52 & 16 & 24 \end{array}\right]$$

$$R_1 \leftarrow R_1 + \frac{17}{2}R_3$$

$$R_2 \leftarrow R_2 - 4R_3$$

$$R_4 \leftarrow R_4 - 52R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{3} & \frac{-35}{3} \\ 0 & 1 & 0 & \frac{-1}{3} & \frac{17}{3} \\ 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & \frac{-4}{3} & \frac{20}{3} \end{array}\right]$$

$$R_4 \leftarrow \frac{-3}{4}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{3} & \frac{-35}{3} \\ 0 & 1 & 0 & \frac{-1}{3} & \frac{17}{3} \\ 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 & -5 \end{array}\right]$$

$$R_1 \leftarrow R_1 - \frac{1}{3}R_4$$

$$R_2 \leftarrow R_2 + \frac{1}{3}R_4$$

$$R_3 \leftarrow R_3 - \frac{1}{3}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -10 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{array}\right]$$

$$x_1 = -10$$

$$x_2 = 4$$

$$x_3 = 2$$

$$x_4 = -5$$

21.

$$2x_1 + x_2 - 3x_3 = 2$$

$$4x_1 + 3x_2 - 2x_3 = -11$$

$$-4x_3 = 12$$

$$-2x_1 + x_2 + 3x_3 = -8$$

$$2x_1 - 2x_2 + 3x_3 = -7$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -3 & 2 \\ 4 & 3 & -2 & -11 \\ 0 & 0 & -4 & 12 \\ -2 & 1 & 3 & -8 \\ 2 & -2 & 3 & -7 \end{array}\right]$$

$$R_1 \leftarrow \frac{1}{2}R_1$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{-3}{2} & 1 \\ 4 & 3 & -2 & -11 \\ 0 & 0 & -4 & 12 \\ -2 & 1 & 3 & -8 \\ 2 & -2 & 3 & -7 \end{array}\right]$$

$$R_2 \leftarrow R_2 - 4R_1$$

$$R_4 \leftarrow R_4 + 2R_1$$

$$R_5 \leftarrow R_5 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{-3}{2} & 1 \\ 0 & 1 & 4 & -15 \\ 0 & 0 & -4 & 12 \\ 0 & 2 & 0 & -6 \\ 0 & -3 & 6 & -9 \end{array}\right]$$

$$R_1 \leftarrow R_1 - \frac{1}{2}R_2$$

$$R_4 \leftarrow R_4 - 2R_2$$

$$R_5 \leftarrow R_5 + 3R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{-7}{2} & \frac{17}{2} \\ 0 & 1 & 4 & -15 \\ 0 & 0 & -4 & 12 \\ 0 & 0 & -8 & 24 \\ 0 & 0 & 18 & -54 \end{array}\right]$$

$$R_3 \leftarrow \frac{-1}{4}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{-7}{2} & \frac{17}{2} \\ 0 & 1 & 4 & -15 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & -8 & 24 \\ 0 & 0 & 18 & -54 \end{array}\right]$$

$$R_1 \leftarrow R_1 + \frac{7}{2}R_3$$

$$R_2 \leftarrow R_2 - 4R_3$$

$$R_4 \leftarrow R_4 + 8R_3$$

$$R_5 \leftarrow R_5 - 18R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 = -2$$

$$x_2 = -3$$

$$x_3 = -3$$

22.

$$x_1 - 3x_2 + 2x_3 + 4x_4 - 4x_5 + 4x_6 = 23$$

$$4x_1 - 4x_2 + 2x_3 - 3x_4 - x_5 - 2x_6 = 19$$

$$\left[\begin{array}{cccccc|c} 1 & -3 & 2 & 4 & -4 & 4 & 23 \\ 4 & -4 & 2 & -3 & -1 & -2 & 19 \end{array}\right]$$

$$R_2 \leftarrow R_2 - 4R_1$$

$$\left[\begin{array}{cccccc|c} 1 & -3 & 2 & 4 & -4 & 4 & 23 \\ 0 & 8 & -6 & -19 & 15 & -18 & -73 \end{array}\right]$$

$$R_2 \leftarrow \frac{1}{8}R_2$$

$$\left[\begin{array}{cccccc|c} 1 & -3 & 2 & 4 & -4 & 4 & 23 \\ 0 & 1 & \frac{-3}{4} & \frac{-19}{8} & \frac{15}{8} & \frac{-9}{4} & \frac{-73}{8} \end{array}\right]$$

$$R_1 \leftarrow R_1 + 3R_2$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & \frac{-1}{4} & \frac{-25}{8} & \frac{13}{8} & \frac{-11}{4} & \frac{-35}{8} \\ 0 & 1 & \frac{-3}{4} & \frac{-19}{8} & \frac{15}{8} & \frac{-9}{4} & \frac{-73}{8} \end{array}\right]$$

$$x_1 - \frac{1}{4}x_3 - \frac{25}{8}x_4 + \frac{13}{8}x_5 - \frac{11}{4}x_6 = \frac{-35}{8}$$

$$x_2 - \frac{3}{4}x_3 - \frac{19}{8}x_4 + \frac{15}{8}x_5 - \frac{9}{4}x_6 = \frac{-73}{8}$$

$$x_1 = \frac{-35}{8} + \frac{1}{4}r + \frac{25}{8}s - \frac{13}{8}t + \frac{11}{4}k$$

$$x_2 = \frac{-73}{8} + \frac{3}{4}r + \frac{19}{8}s - \frac{15}{8}t + \frac{9}{4}k$$

$$x_3 = r$$

$$x_4 = s$$

$$x_5 = t$$

$$x_6 = k$$

$$r, s, t, k \in \mathbb{R}$$

23.

$$-2x_1 + 2x_2 + 3x_3 - 4x_4 - 2x_5 = 19$$

$$3x_1 - 3x_2 - 3x_3 - x_4 + 3x_5 = -14$$

$$-2x_2 - 3x_4 - 2x_5 = 19$$

$$\left[\begin{array}{ccccc|c} -2 & 2 & 3 & -4 & -2 & 19 \\ 3 & -3 & -3 & -1 & 3 & -14 \\ 0 & -2 & 0 & -3 & -2 & 19 \end{array}\right]$$

$$R_1 \leftarrow R_1 + R_2$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & -5 & 1 & 5 \\ 3 & -3 & -3 & -1 & 3 & -14 \\ 0 & -2 & 0 & -3 & -2 & 19 \end{array}\right]$$

$$R_2 \leftarrow R_2 - 3R_1$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & -5 & 1 & 5 \\ 0 & 0 & -3 & 14 & 0 & -29 \\ 0 & -2 & 0 & -3 & -2 & 19 \end{array}\right]$$

$$R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & -5 & 1 & 5 \\ 0 & -2 & 0 & -3 & -2 & 19 \\ 0 & 0 & -3 & 14 & 0 & -29 \end{array}\right]$$

$$R_2 \leftarrow \frac{-1}{2}R_2$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & -5 & 1 & 5 \\ 0 & 1 & 0 & \frac{3}{2} & 1 & \frac{-19}{2} \\ 0 & 0 & -3 & 14 & 0 & -29 \end{array}\right]$$

$$R_1 \leftarrow R_1 + R_2$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & \frac{-7}{2} & 2 & \frac{-9}{2} \\ 0 & 1 & 0 & \frac{3}{2} & 1 & \frac{-19}{2} \\ 0 & 0 & -3 & 14 & 0 & -29 \end{array}\right]$$

$$R_3 \leftarrow \frac{-1}{3}R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & \frac{-7}{2} & 2 & \frac{-9}{2} \\ 0 & 1 & 0 & \frac{3}{2} & 1 & \frac{-19}{2} \\ 0 & 0 & 1 & \frac{-14}{3} & 0 & \frac{29}{3} \end{array}\right]$$

$$x_1 - \frac{7}{2}x_4 + 2x_5 = \frac{-9}{2}$$

$$x_2 + \frac{3}{2}x_4 + x_5 = \frac{-19}{2}$$

$$x_3 - \frac{14}{3}x_4 = \frac{29}{3}$$

$$x_1 = \frac{-9}{2} + \frac{7}{2}r - 2s$$

$$x_2 = \frac{-19}{2} - \frac{3}{2}r - s$$

$$x_3 = \frac{29}{3} + \frac{14}{3}r$$

$$x_4 = r$$

$$x_5 = s$$

$$r, s \in \mathbb{R}$$

24.

$$-2x_2 + 3x_3 + x_4 = 16$$
$$3x_1 + x_2 - x_3 + 2x_4 = 26$$
$$4x_1 - 3x_2 + 3x_3 - 3x_4 = 15$$
$$2x_1 - 2x_3 - 2x_4 = -12$$

$$\left[\begin{array}{cccc|c} 0 & -2 & 3 & 1 & 16 \\ 3 & 1 & -1 & 2 & 26 \\ 4 & -3 & 3 & -3 & 15 \\ 2 & 0 & -2 & -2 & -12 \end{array}\right]$$

$$R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 3 & 1 & -1 & 2 & 26 \\ 0 & -2 & 3 & 1 & 16 \\ 4 & -3 & 3 & -3 & 15 \\ 2 & 0 & -2 & -2 & -12 \end{array}\right]$$

$$R_1 \leftarrow R_1 - R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 4 & 38 \\ 0 & -2 & 3 & 1 & 16 \\ 4 & -3 & 3 & -3 & 15 \\ 2 & 0 & -2 & -2 & -12 \end{array}\right]$$

$$R_3 \leftarrow R_3 - 4R_1$$
$$R_4 \leftarrow R_4 - 2R_1$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 4 & 38 \\ 0 & -2 & 3 & 1 & 16 \\ 0 & -7 & -1 & -19 & -137 \\ 0 & -2 & -4 & -10 & -88 \end{array}\right]$$

$$R_2 \leftarrow R_2 - \frac{3}{2}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 4 & 38 \\ 0 & 1 & 9 & 16 & 148 \\ 0 & -7 & -1 & -19 & -137 \\ 0 & -2 & -4 & -10 & -88 \end{array}\right]$$

$$R_1 \leftarrow R_1 - R_2$$
$$R_3 \leftarrow R_3 + 7R_2$$
$$R_4 \leftarrow R_4 + 2R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -8 & -12 & -110 \\ 0 & 1 & 9 & 16 & 148 \\ 0 & 0 & 62 & 93 & 899 \\ 0 & 0 & 14 & 22 & 208 \end{array}\right]$$

$$R_3 \leftarrow \frac{1}{62}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -8 & -12 & -110 \\ 0 & 1 & 9 & 16 & 148 \\ 0 & 0 & 1 & \frac{3}{2} & \frac{29}{2} \\ 0 & 0 & 14 & 22 & 208 \end{array}\right]$$

$$R_1 \leftarrow R_1 + 8R_3$$
$$R_2 \leftarrow R_2 - 9R_3$$
$$R_4 \leftarrow R_4 - 14R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & \frac{5}{2} & \frac{35}{2} \\ 0 & 0 & 1 & \frac{3}{2} & \frac{29}{2} \\ 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

$$R_2 \leftarrow R_2 - \frac{5}{2}R_4$$
$$R_3 \leftarrow R_3 - \frac{3}{2}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

$$x_1 = 6$$
$$x_2 = 5$$
$$x_3 = 7$$
$$x_4 = 5$$

25.

$$4x_1 - 4x_2 = -64$$
$$-3x_1 + 4x_2 = 56$$
$$x_1 - x_2 = -16$$
$$-2x_1 + 4x_2 = 48$$
$$2x_1 + x_2 = -8$$

$$\left[\begin{array}{cc|c} 4 & -4 & -64 \\ -3 & 4 & 56 \\ 1 & -1 & -16 \\ -2 & 4 & 48 \\ 2 & 1 & -8 \end{array}\right]$$

$$R_1 \leftarrow R_1 + R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & -8 \\ -3 & 4 & 56 \\ 1 & -1 & -16 \\ -2 & 4 & 48 \\ 2 & 1 & -8 \end{array}\right]$$

$$R_2 \leftarrow R_2 + 3R_1$$
$$R_3 \leftarrow R_3 - R_1$$
$$R_4 \leftarrow R_4 + 2R_1$$
$$R_5 \leftarrow R_5 - 2R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 4 & 32 \\ 0 & -1 & -8 \\ 0 & 4 & 32 \\ 0 & 1 & 8 \end{array}\right]$$

$$R_2 \leftarrow \frac{1}{4}R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 8 \\ 0 & -1 & -8 \\ 0 & 4 & 32 \\ 0 & 1 & 8 \end{array}\right]$$

$$R_3 \leftarrow R_3 + R_2$$

$$R_4 \leftarrow R_4 - 4R_2$$

$$R_5 \leftarrow R_5 - R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$x_1 = -8$$

$$x_2 = 8$$