Question 1. *Prove using mathematical induction that for all $n \geq 1$,*

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}.$$

Question 2. *Use the Principle of Mathematical Induction to verify that, for n any positive integer, $6^n - 1$ is divisible by 5.*

Question 3. *Verify that for all $n \geq 1$, the sum of the squares of the first $2n$ positive integers is given by the formula*

$$1^2 + 2^2 + 3^2 + \cdots + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}$$

Question 4. *Consider the sequence of real numbers defined by the relations*

$$x_1 = 1 \text{ and } x_{n+1} = \sqrt{1 + 2x_n} \text{ for } n \geq 1.$$

Use the Principle of Mathematical Induction to show that $x_n < 4$ for all $n \geq 1$.

Question 5. *Show that $n! > 3^n$ for $n \geq 7$.*

Question 6. *Let $p_0 = 1$, $p_1 = \cos\theta$ (for θ some fixed constant) and $p_{n+1} = 2p_1p_n - p_{n-1}$ for $n \geq 1$. Use an extended Principle of Mathematical Induction to prove that $p_n = \cos(n\theta)$ for $n \geq 0$.*

Question 7. *Consider the famous Fibonacci sequence $\{x_n\}_{n=1}^{\infty}$, defined by the relations $x_1 = 1$, $x_2 = 1$, and $x_n = x_{n-1} + x_{n-2}$ for $n \geq 3$.*

(a) Compute x_{20}.

(b) Use an extended Principle of Mathematical Induction in order to show that for $n \geq 1$,

$$x_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right].$$

(c) Use the result of part (b) to compute x_{20}.