UNIVERSITY OF MANITOBA

DATE: June 25, 2009 — FINAL EXAMINATION
PAPER # 64 — TITLE PAGE
COURSE: MATH 1210 — TIME: 2 hours
EXAMINATION: Classical and Linear Algebra — EXAMINER: M. Davidson

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FAMILY NAME: (Print in ink) ______________________

GIVEN NAME(S): (Print in ink) ______________________

STUDENT NUMBER: ____________

SEAT NUMBER: ____________

SIGNATURE: (in ink) ______________________
(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 10 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	30	
2	10	
3	18	
4	14	
5	12	
6	8	
7	8	
Total:	100	

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1. The following are short answer questions.

[3] (a) What is the polar form of $-2\sqrt{3}+2i$?

[3] (b) Find the cross product of the vectors $\overrightarrow{u} = [1,-1,3]$ and $\overrightarrow{v} = [4,-6,-2]$.

[4] (c) Use the adjoint to find the inverse of the matrix $A = \begin{pmatrix} 7 & 4 \\ -2 & 3 \end{pmatrix}$.

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[3] (d) Let T be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by $T(\overrightarrow{x}) = A\overrightarrow{x}$ where $A = \begin{pmatrix} 3 & -2 & 2 \\ 2 & -5 & 10 \\ 1 & -4 & 8 \end{pmatrix}$. Show that $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ is an eigenvector of T and find the associated eigenvalue.

[3] (e) Simplify the expression $\overline{4+3i} + \dfrac{2-3i}{5-2i}$ into Cartesian form.

[3] (f) Write the following in sigma notation (do not evaluate) :
$1 - 3 + 5 - 7 + \cdots + (4n-3) - (4n-1)$.

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[2] (g) Suppose the coefficient matrix of a homogeneous system is a 5×7 matrix of rank 4. How many linearly independent basic solutions will it have?

[2] (h) Are the vectors $\{[1, 3, 7], [3, 2, -5], [4, -1, 6], [2, -7, 2]\}$ linearly dependent or linearly independent. Justify your answer.

[7] (i) Find, in exponential form, all solutions to $z^3 = -8i$

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[10] 2. Use mathematical induction to show that for all $n \geq 1$ that

$$2 + 5 + 8 + \ldots + (6n - 1) = n(6n + 1)\,.$$

[18] 3. Consider the polynomial $P(x) = 3x^4 - 13x^3 + 31x^2 - 39x + 10$.

(a) Apply Descartes rules of signs to $P(x)$. Be specific about what information it gives.

(b) Apply the bound theorem to $P(x)$. Be specific about what information it gives (Write it in a sentence).

(c) What are the possible rational roots of $P(x)$? Include any information from part a and/or part b.

(d) Find all roots of $P(x) = 3x^4 - 13x^3 + 31x^2 - 39x + 10$

[14] 4. Consider the vectors:

$$\overrightarrow{u_1} = [1,2,3] \qquad \overrightarrow{u_2} = [-1,3,2] \qquad \overrightarrow{u_3} = [4,2,5] \qquad \overrightarrow{v} = [5,3,6]$$

(a) Show that the vectors $\{\overrightarrow{u_1}, \overrightarrow{u_2}, \overrightarrow{u_3}\}$ are linearly independent? (Justify your answer.)

(b) Use Cramer's rule to solve $\overrightarrow{v} = c_1\,\overrightarrow{u_1} + c_2\,\overrightarrow{u_2} + c_3\,\overrightarrow{u_3}$ for the variables c_1, c_2 and c_3.

(c) Write $\overrightarrow{v}$ as a linear combination of $\{\overrightarrow{u_1}, \overrightarrow{u_2}, \overrightarrow{u_3}\}$.

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[12] 5. (a) Using the *direct method*, find the inverse of the matrix $A = \begin{pmatrix} 2 & 2 & -1 \\ 3 & -2 & 4 \\ 1 & -1 & 2 \end{pmatrix}$.

(No other method will be accepted)

(b) Use the information from part a to find a solution to :

$$\begin{array}{rcrcrcr} 2x_1 & + & 2x_2 & - & x_3 & = & 2 \\ 3x_1 & - & 2x_2 & + & 4x_3 & = & -1 \\ x_1 & - & x_2 & + & 2x_3 & = & 3 \end{array}$$

(No other method will be accepted)

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[8] 6. Let T be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that is defined by $T(\tilde{x}) = A\tilde{x}$ where $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 3 & -1 \\ 2 & -1 & 2 \end{pmatrix}$. Find all eigenvalues of T.

(DO NOT SOLVE FOR THE EIGENVECTORS)

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[8] 7. Let T be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that is defined by $T(\tilde{x}) = A\tilde{x}$ where $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 3 \end{pmatrix}$. T has eigenvalues $\lambda = 2, 1$ (One of the eigenvalues has multiplicity 2). Find all eigenvectors associated with each eigenvalue of T.