UNIVERSITY OF MANITOBA

DATE: June 27, 2013 FINAL EXAMINATION

TITLE PAGE

EXAMINATION: Tech. of Classical and Linear Algebra TIME: 2 hours

COURSE: MATH 1210 EXAMINER: Harland

NAME: (in ink) ____________________

STUDENT NUMBER: ____________________

SIGNATURE: (in ink) ____________________

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 11 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but **be careful not to loosen the staple.**

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 105 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	8	
2	7	
3	6	
4	7	
5	7	
6	5	
7	7	
8	6	
9	13	
10	11	
11	8	
12	6	
13	14	
Total:	105	

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[8] 1. Use mathematical induction to prove that 3 divides $n^3 - n$, for all positive integers $n \geq 2$.

[7] 2. Find the remainder (in Cartesian form) when the polynomial $P(x) = x^8 + ix + 2$ is divided by $x - (1 + i)$.

[6] 3. Find the equation of the plane containing the point $P(2, 3, -2)$ and the line with symmetric equations

$$\frac{x-1}{6} = \frac{y+1}{2} = z - 3.$$

4. Let $P(x)$ be the polynomial $8x^3 - ax^2 + 3$.

[3] (a) List the possible rational solutions of $P(x) = 0$.

[4] (b) Find the value(s) of a such that $2x - 3$ is a factor of $P(x)$.

[7] 5. Solve the following system of linear equations by Gauss-Jordan Elimination:

$$\begin{aligned} x + 2y + 5z &= 5 \\ x + 3y + 3z &= 4 \\ y - 2z &= -1 \end{aligned}$$

[5] 6. Find the determinant of $\begin{pmatrix} 2 & 4 & 6 & 8 \\ 2 & 4 & 6 & 13 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 5 & 6 \end{pmatrix}$.

7. Let A, B be 4×4 matrices with $|A| = -2$ and $|B| = 3$.

[4] (a) Find $\left|2B^{-1}AB^T\right|$.

[3] (b) Find $|adj(A)|$.

[6] 8. For the system of linear equations,

$$\begin{aligned} x + 3y + 3z &= 4 \\ x + 2y + 5z &= 6 \\ y - z &= 0, \end{aligned}$$

use Cramer's Rule to solve for y. (Do not solve for x or z)

[7] 9. (a) Are the following vectors linearly independent or linearly dependent? If they are linearly dependent, express one vector as a linear combination of the others.

$$\mathbf{u} = \langle 1, 0, 2, 0 \rangle, \quad \mathbf{v} = \langle 0, 2, -3, 0 \rangle, \quad \mathbf{w} = \langle 2, 4, -2, 0 \rangle$$

[2] (b) Are the following vectors linearly dependent or independent? Justify your answer.

$$\mathbf{u} = \langle -1, 0, 2 \rangle, \quad \mathbf{v} = \langle -1, 2, 3 \rangle$$

[4] (c) Are the following vectors linearly dependent or independent? Justify your answer.

$$\mathbf{u} = \langle 1, 0, 2\rangle, \quad \mathbf{v} = \langle -1, 3, 0\rangle, \quad \mathbf{w} = \langle 0, 3, 4\rangle$$

10. For the matrix $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -1 \\ 0 & 1 & 2 \end{pmatrix}$:

[6] (a) Use row reduction to find A^{-1} if it exists.

[2] (b) Use the information from part (a) to solve the system

$$\begin{aligned} x + 2y - z &= 1 \\ 2x + 4y - z &= 1 \\ y + 2z &= 1. \end{aligned}$$

[3] (c) Use the information from part (a) to solve the system

$$\begin{aligned} x + 2y &= 1 \\ 2x + 4y + z &= 1 \\ -x - y + 2z &= 1. \end{aligned}$$

[8] 11. The matrix

$$A = \begin{pmatrix} 1 & 3 & -1 \\ 2 & 4 & -1 \\ 1 & 1 & 2 \end{pmatrix}$$

has determinant of 4 and the adjoint is

$$adj(A) = \begin{pmatrix} x & 5 & 1 \\ 3 & -1 & y \\ -2 & 2 & -2 \end{pmatrix}.$$

Solve for x, y, and A^{-1}.

12. Let T be the linear transformation

$$T: \begin{array}{ll} v_1' & = v_1 - 4v_2 \\ v_2' & = 2v_1 - 3v_2 \end{array}.$$

[2] (a) Is T linear? Justify your answer.

[1] (b) Find $T\langle 2, 5\rangle$.

[3] (c) Find $\mathbf{v}$ such that $T(\mathbf{v}) = \langle 1, 2\rangle$.

13. Let A be the matrix $\begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & -1 \\ 0 & -3 & 4 \end{pmatrix}$.

[6] (a) Find the eigenvalues of A. (Note: is may be helpful to use that $\lambda = 1$ is an eigenvalue)

[6] (b) Find the eigenvectors corresponding to $\lambda = 1$.

[2] (c) A 4×4 matrix B has eigenvalues $1, 2, 3, 4$. Solve $B\mathbf{x} = -3\mathbf{x}$. Justify your answer.