UNIVERSITY OF MANITOBA

DATE: June 25, 2015 FINAL EXAMINATION

TITLE PAGE

EXAMINATION: Techniques of Classical and Linear Algebra TIME: 2 hours

COURSE: MATH 1210 EXAMINER: Harland

FAMILY NAME (Write in Capital Letters): ______________________________

GIVEN NAME (Write in Capital Letters): ______________________________

STUDENT NUMBER: ______________________________

SIGNATURE: (in ink) ____

(I understand that cheating is a serious offense.)

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 11 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. **DO NOT REMOVE THE SCRAP PAPER.**

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 110 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	8	
2	7	
3	6	
4	12	
5	7	
6	5	
7	7	
8	6	
9	13	
10	11	
11	8	
12	6	
13	14	
Total:	110	

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[8] 1. Use the principle of mathematical induction to prove that 3 divides $n^3 - n$, for all positive integers $n \geq 2$.

[7] 2. Calculate the remainder (in Cartesian form) when the polynomial $P(x) = x^{10} + ix + 2$ is divided by $x - (1 + i)$.

[6] 3. Determine an equation of the plane containing the point $P(2, 3, -2)$ and the line with symmetric equations

$$\frac{x-1}{6} = \frac{y+1}{2} = \frac{z-3}{-2}.$$

4. Let $P(x)$ be the polynomial $8x^3 - ax^2 + 25$.

[3] (a) List the possible rational solutions of $P(x) = 0$.

[4] (b) Calculate the value(s) of a such that $2x - 5$ is a factor of $P(x)$.

[5] (c) Using the value(s) of a in part (b), find all roots of $P(x)$.

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[7] 5. Solve the following system of linear equations by Gauss-Jordan Elimination:

$$\begin{aligned} x + 2y + 5z &= 5 \\ x + 3y + 3z &= 4 \\ y - 2z &= -1 \end{aligned}$$

[5] 6. Calculate the determinant of $\begin{pmatrix} 2 & 4 & 6 & 8 \\ 2 & 4 & 6 & 13 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 5 & 6 \end{pmatrix}$.

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7. Let A, B be 3×3 matrices with $|A| = -2$ and $|B| = 3$.

[4] (a) Calculate $\left|3B^{-1}A^2B^T\right|$.

[3] (b) Calculate $|adj(A)|$.

[6] 8. For the system of linear equations,

$$\begin{aligned} x + 3y + 3z &= 4 \\ x + 2y + 5z &= 6 \\ y - z &= 0, \end{aligned}$$

use Cramer's Rule to solve for z. (Do not solve for x or y)

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[7] 9. (a) Are the following vectors linearly independent or linearly dependent? If they are linearly dependent, express one vector as a linear combination of the others.

$$\mathbf{u} = \langle 1, 0, 2, 0 \rangle, \quad \mathbf{v} = \langle 0, 2, -3, 0 \rangle, \quad \mathbf{w} = \langle 2, 4, -2, 0 \rangle$$

[2] (b) Are the following vectors linearly dependent or independent? Justify your answer.

$$\mathbf{u} = \langle -1, 0, 2 \rangle, \quad \mathbf{v} = \langle -1, 2, 3 \rangle$$

[4] (c) Are the following vectors linearly dependent or independent? Justify your answer.

$$\mathbf{u} = \langle 1, 0, 2\rangle, \quad \mathbf{v} = \langle -1, 3, 0\rangle, \quad \mathbf{w} = \langle 0, 3, 4\rangle$$

10. For the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{pmatrix}$:

[6] (a) Use row reduction (the direct method) to calculate A^{-1} if it exists.

[2] (b) Use the information from part (a) to solve the system

$$\begin{aligned} x + 2y + 3z &= 2 \\ 2x + 4y + 5z &= 1 \\ y + 2z &= 1. \end{aligned}$$

[3] (c) Use the information from part (a) to solve the system

$$\begin{aligned} x + 2y &= 2 \\ 2x + 4y + z &= 1 \\ 3x + 5y + 2z &= 1. \end{aligned}$$

[8] 11. The matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 4 & 5 \\ 5 & 1 & 2 \end{pmatrix}$$

has determinant of -11 and the adjoint is

$$\mathrm{adj}(A) = \begin{pmatrix} 3 & -1 & -2 \\ 17 & -13 & y \\ x & 9 & -4 \end{pmatrix}.$$

Solve for x, y, and A^{-1}.

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12. Let T be the linear transformation

$$T : \begin{array}{ll} v_1' & = v_1 - 4v_2 \\ v_2' & = 2v_1 - 3v_2 + 1 \end{array} .$$

[2] (a) Is T linear? Justify your answer.

[1] (b) Calculate $T\langle 1, 2\rangle$.

[3] (c) Determine $\mathbf{v}$ such that $T(\mathbf{v}) = \langle 0, 0\rangle$.

13. Let A be the matrix $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 4 & 6 \\ 0 & 5 & 11 \end{pmatrix}$.

[6] (a) Calculate the eigenvalues of A. (Note: it may be helpful to use that $\lambda = 1$ is an eigenvalue)

[6] (b) Calculate the eigenvectors corresponding to $\lambda = 1$.

[2] (c) A 4×4 matrix B has eigenvalues $1, 2, 3, 4$. Solve $B\mathbf{x} = -3\mathbf{x}$. Justify your answer.