University of Manitoba
Department of Mathematics

VECTOR-GEOMETRY REVIEW EXERCISES

1. Find an equation for each of the following planes:
(a) through the point $P(0,5,6)$ and perpendicular to the vector $4\mathbf{i}+2\mathbf{j}-\mathbf{k}$.
(b) through $P\,(2,-1,3)$ and perpendicular to $L: x=-1+2t,\ y=1+3t,\ z=-4t$.
(c) through $P\,(1,-2,-1)$ and parallel to the plane $3x+2y-z+4=0$.
(d) containing $A\,(1,-2,1)\,, B\,(2,0,3)$ and $C\,(0,1,-1)$.
(e) containing $A\,(3,-1,2)\,, B\,(1,2,-1)$ and $C\,(2,3,1)$.
(f) containing $L_1: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + s \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$, $\quad L_2: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.
(g) containing $L_1: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\quad L_2: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$.
(h) containing $L_1: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + s \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$, $\quad L_2: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ $\quad (L_1 \,\|\, L_2)$.
(i) containing $P\,(3,-1,2)$ and $L: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$.

2. Find equations for each of the following lines:
(a) through $P\,(1,0,-1)$ with direction $\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$.
(b) through $P\,(4,0,0)$ parallel to $2\mathbf{i}-\mathbf{j}-2\mathbf{k}$.
(c) through $P\,(2,-1,3)$ parallel to $L: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + s \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$.
(d) through $A\,(1,2,-1)$ and $B\,(3,-1,2)$.
(e) through $A\,(2,3,4)$ and $B\,(-1,-3,2)$.
(f) through $P\,(-2,3,1)$ and perpendicular to the plane $2x+3y+z=3$.
(g) of intersection of the planes $3\,(x+2)-2\,(y-1)+2\,(z+1)=0$ and $(x+2)+2\,(y-1)-3\,(z+1)=0$.
(h) of intersection of the planes $2\,(x-1)+3\,(y+1)-4\,(z-2)=0$ and $3\,(x-1)-4\,(y+1)+2\,(z-2)=0$.
(i) through $P\,(3,-1,2)$ that is perpendicular to and intersects $L: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.
(j) through $P\,(0,2,4)$ that is perpendicular to and intersects $L: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + s \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$.
(k) through $P\,(2,1,3)$ that is perpendicular to $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and to $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$.

3. Find the following distances:
(a) from $P\,(2,1,-1)$ to the plane $x-2y+2z+5=0$.
(b) from $P\,(-1,3,2)$ to the plane $2x-3y+4z-5=0$.
(c) from $A\,(1,-2,3)$ to BC where B is $(3,1,2)$ and C is $(2,3,-1)$ (Also find the area of ΔABC).
(d) from $A\,(2,-1,1)$ to BC where B is $(3,2,-1)$ and C is $(-1,3,2)$ (Also find the area of ΔABC).
(e) between $L_1: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix} + s \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$ and $L_2: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix} + t \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$ $(L_1 \,\|\, L_2)$.

(f) between $L_1 : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $L_2 : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix} + t \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}$.

(g) between $L_1 : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} + s \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$ and $L_2 : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} 3 \\ 5 \\ -2 \end{bmatrix}$.

(h) between the parallel planes $3(x-1)+2(y-1)-6(z+1)=0$ and $3(x+2)+2y-6(z-1)=0$.

(i) between the parallel planes $x+2y-3z=1$ and $x+2y-3z=5$.

4. Find the following points of intersection:

(a) of $L : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ with the plane $x+y-3z=7$.

(b) of $L :$ $\mathrm{x} = 2-t, \quad y = 1+2t, \quad z = 3+t$ with the plane $2x-y+z=2$.

ANSWERS

1. (a) $4x+2y-z=4$, (b) $2x+3y-4z+11=0$, (c) $3x+2y-z=0$, (d) $2x-z=1$, (e) $9x+y-5z=16$, (f) $2x-2y-z=4$, (g) $2x-y-z+3=0$, (h) $x-4y-10z=6$, (i) $2x-2y-z=6$.

2. (a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$, (b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$, (c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + s \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$,
(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + \mathrm{s} \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$, (e) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} + \mathrm{s} \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$, (f) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} + \mathrm{s} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$,
(g) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix} + \mathrm{s} \begin{bmatrix} 2 \\ 11 \\ 8 \end{bmatrix}$, (h) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + \mathrm{s} \begin{bmatrix} 10 \\ 16 \\ 17 \end{bmatrix}$, (i) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + \mathrm{s} \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$,
(j) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} + \mathrm{s} \begin{bmatrix} 29 \\ 1 \\ -22 \end{bmatrix}$, (k) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + \mathrm{s} \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}$.

3. (a) 1. (b) $8/\sqrt{29}$. (c) $\frac{1}{2}\sqrt{42}, \frac{7}{2}\sqrt{3}$. (d) $\frac{3}{26}\sqrt{910}, \frac{3}{2}\sqrt{35}$. (e) $4\sqrt{2}$. (f) $\frac{13}{10}\sqrt{6}$. (g) $\frac{68}{3\sqrt{78}}$. (h) $\frac{23}{7}$. (i) $\frac{2}{7}\sqrt{14}$.

4. (a) $\left(\frac{31}{10}, -\frac{3}{10}, -\frac{7}{5}\right)$, (b) $\left(\frac{2}{3}, \frac{11}{3}, \frac{13}{3}\right)$.

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