UNIVERSITY OF MANITOBA

DATE: April 18, 2008 — FINAL EXAMINATION
PAPER # 369 — TITLE PAGE
COURSE: MATH 1210 — TIME: 2 hours
EXAMINATION: Classical and Linear Algebra — EXAMINER: Davidson, Thomas

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FAMILY NAME: (Print in ink) _____

GIVEN NAME(S): (Print in ink) _____

STUDENT NUMBER: _____

SEAT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

- ❑ A01 Dr. Robert Thomas
- ❑ A02 Dr. Michelle Davidson

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 7 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 75 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	31	
2	12	
3	5	
4	12	
5	15	
Total:	75	

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1. The following are short answer questions.

[4] (a) What do Descartes' rules of signs imply about the polynomial $P(x) = 7x^5 - 5x^4 + 3x^3 + 17x^2 - 2x + 16$?

[3] (b) Find an equation of the line that contains the point $(9, 3, -7)$ and is parallel to the line $\ell : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 6 \end{bmatrix} + t \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$.

[3] (c) Use the adjoint to find the inverse of the matrix $A = \begin{pmatrix} -2 & 4 \\ -3 & 7 \end{pmatrix}$.

[2] (d) Are the vectors $\{(1, 1), (-3, 12), (6, -7)\}$ linearly dependent or linearly independent? Justify your answer.

[2] (e) Write $-4\sqrt{3}+4i$ in polar form.

[3] (f) Use the remainder theorem to find the remainder when the polynomial $P(x)=2x^3+5x^2-x+2$ is divided by $x+2i$.

[2] (g) Write the following in sigma notation (do not evaluate) :
$-1+4-9+16-25+36-49+64-81$

[2] (h) Given $A=\begin{pmatrix}1 & -2\\ 3 & 1\end{pmatrix}$ and $B=\begin{pmatrix}2 & -4 & 2\\ 1 & 6 & -1\end{pmatrix}$ then $AB=\begin{pmatrix}0 & -16 & 4\\ 7 & -6 & 5\end{pmatrix}$.
What is B^TA^T?

[3] (i) Find the point of intersection of the line $\ell : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 6 \end{bmatrix} + t \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$ and the plane $3x - 2y + 4z = 15$.

[4] (j) Are the vectors $\{(1, -7, 3), (2, 1, 1), (5, 7, 1)\}$ linearly dependent or linearly independent? Justify your answer.

[3] (k) If $z = 2 - 2i$, what is z^8? Simplify fully. (hint: this may be easier using DeMoivre's theorem.)

[12] 2. Use mathematical induction to show that $5^{2n} - 1$ is divisible by 8 for all $n \geq 1$.

[5] 3. Use Cramer's rule to find the solution to the system of equations:

$$\begin{aligned} 2x &- y = 8 \\ 3x &+ 6y = 3 \end{aligned}$$

[12] 4. (a) Use the direct method to find the inverse of the matrix $A = \begin{pmatrix} 1 & 5 & -4 \\ -2 & -3 & 6 \\ 1 & 2 & -3 \end{pmatrix}$.

(b) Use the information from part (a) to find the solution to :

$$\begin{array}{rcrcrcr} x_1 & + & 5x_2 & - & 4x_3 & = & -1 \\ -2x_1 & - & 3x_2 & + & 6x_3 & = & 2 \\ x_1 & + & 2x_2 & - & 3x_3 & = & 4 \end{array}$$

[15] 5. Let T be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by $T(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{pmatrix} 2 & 4 & -4 \\ 0 & -1 & 3 \\ 0 & -4 & 6 \end{pmatrix}$. Find all eigenvalues of T. Find all eigenvectors associated with each eigenvalue of T.