MATH1210 Test #1 12 February 2009 Time : 60 minute
Instructor: (check one) [ ] Berry (A01) [ ] Borgersen (A02)

NAME: ________________________________ **ID#:** _________________

[10 marks] Use the Principle of Mathematical Induction to show that the statement

$P_n : \sum_{\ell=n}^{2n} \ell = \frac{3n(n+1)}{2}$ is true for n any positive integer.

NOTE CAREFULLY that the summation index runs from n to 2n.

(SHOW ALL YOUR WORK!)

[10 marks] Consider the complex number $z = \left(1 + i - \dfrac{1}{1-i}\right)$.

(a) Express z in **Cartesian form**, simplifying your answer as far as possible.

(b) Express z in **exponential form**, indication clearly its modulus and the **principal value** of its argument.

(c) Express $\bar{z}$ in **exponential form**.

(d) Use the above results to compute $\bar{z}^3\left(\dfrac{1}{z}\right)$, expressing your answer in **Cartesian form**.

[10 marks] Consider the complex number $z = 16\left(-\sqrt{3} + i\right)$.

(a) Express z in exponential form.

(b) Find the **modulus** and **principal value of the argument** of each of the **fifth** roots of $z = 16\left(-\sqrt{3} + i\right)$.

[10 marks] Consider the real polynomial $P(x) = x^3 - 7x^2 + 17x - 20$.

(a) Find the remainder when $P(x)$ is divided $(x + 2i)$.

(b) Use Descarte's Rule of signs to determine the maximum number of negative real zeros of $P(x)$.

(c) Use the rational roots theorem to list all the possible rational roots of $P(x) = 0$.

(d) Show that $P(x)$ may be written in the form $P(x) = (x-4)\,Q(x)$ in which $Q(x)$ is an **irreducible real quadratic** factor.

(e) Express $P(x)$ as the product of **linear factors** only.

Problem	1	2	4	4	Total
MARK					
Possible	10	10	10	10	40