UNIVERSITY OF MANITOBA

DATE: April 13, 2010 FINAL EXAMINATION
PAPER # 64 TITLE PAGE
COURSE: MATH 1210 TIME: 2 hours
EXAMINATION: Classical and Linear Algebra EXAMINER: Garba/Mohammed

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FAMILY NAME: (Print in ink) ______________________________

GIVEN NAME(S): (Print in ink) ______________________________

STUDENT NUMBER: ______________

SEAT NUMBER: ______________

SIGNATURE: (in ink) ______________________________
(I understand that cheating is a serious offense)

Please indicate your instructor by placing a check mark in the appropriate box below.

☐ MWF 9:30 - 10:20 S. Garba. ☐ MWF 1:30 - 2:20 T. Mohammed.

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page and 12 pages of questions, which includes 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	35	
2	10	
3	10	
4	8	
5	10	
6	12	
7	6	
8	9	
Total:	100	

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1. The following are short answer questions.

[3] (a) Use the remainder theorem to find the remainder when the polynomial $P(x) = x^3 - 3x^2 + 5x - 3$ is divided by $x - 2i$.

[4] (b) Find all values of k such that the set of vectors $\{(1, k, 2), (2, 1, 4), (k, 4, 6)\}$ is a linearly dependent set.

[3] (c) Find the value of $\cos(\theta)$ where θ is the angle between the vectors $\overrightarrow{u} = [1, -1, 3]$ and $\overrightarrow{v} = [3, -2, -3]$.

[2] (d) Given a linear transformation T such that $T(\hat{\imath}) = (4, -5, 3)$, $T(\hat{\jmath}) = (1, -1, 2)$, and $T(\hat{k}) = (-2, 1, 3)$, what is the value of $T(4, -1, 1)$?

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[4] (e) What does Descartes' rule of signs imply about the polynomial $P(x) = 5x^4 - 4x^3 + 2x^2 + 7x - 13$?

[2] (f) Write the following in sigma (summation) notation (do not evaluate) : $1 - 9 + 25 - 49 + 81 - 121 + 169 - 225$

[3] (g) Simplify the expression $(\overline{2+5i}) + \left(\dfrac{3-i}{1-i}\right)$ into Cartesian form.

[3] (h) Find the exponential form of $z = 2\cos(\frac{\pi}{3}) - 2i\sin(\frac{2\pi}{3})$.

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[3] (i) Given $\displaystyle\sum_{j=1}^{n}(5j-3) = \frac{n(5n-1)}{2}$ evaluate $\displaystyle\sum_{i=5}^{10}(5i-3)$.

[3] (j) Let T be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by $T(\overrightarrow{x}) = A\overrightarrow{x}$ where $A = \begin{pmatrix} 3 & -2 & 2 \\ 2 & -5 & 10 \\ 1 & -4 & 8 \end{pmatrix}$. Show that $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of T and find the associated eigenvalue.

[5] (k) Given that two of the roots of $P(x) = x^4 - 4x^3 + 6x^2 - 4x - 15$ are 3 and $1-2i$, write $P(x)$ as the product of real irreducible quadratic and real linear terms.

[10] 2. Use mathematical induction to show that for all $n \geq 1$ that

$$13 \text{ divides } 3^{3n} - 1\,.$$

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[10] 3. Find, in exponential form, all solutions to $z^5 = -4 + 4i$. The arguments should be express in their principle value.

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[8] 4. Find an equation, in standard form, of the plane that passes through the line

$$x = 5 + 3t \ ; \ y = 1 + 3t \ ; \ z = -t \, ,$$

and the line

$$x = 4 + 3t \ ; \ y = 2 + 3t \ ; \ z = 1 - t \, .$$

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[10] 5. Show that the following vectors are linearly dependent by writing one as a linear combination of the others.

$$\overrightarrow{u_1} = [1, 3, -1] \qquad \overrightarrow{u_2} = [2, 5, 1] \qquad \overrightarrow{u_3} = [2, 3, -1] \qquad \overrightarrow{u_4} = [1, 1, -1]$$

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[12] 6. (a) Using the *adjoint method*, find the inverse of the matrix $A = \begin{pmatrix} 5 & 1 & -2 \\ 3 & -2 & 1 \\ -1 & 4 & -3 \end{pmatrix}$.

(No other method will be accepted)

(b) Use the information from part a to find a solution to :

$$\begin{array}{rcrcrcr} 5x_1 & + & x_2 & - & 2x_3 = & 3 \\ 3x_1 & - & 2x_2 & + & x_3 = & 2 \\ -x_1 & + & 4x_2 & - & 3x_3 = & -2 \end{array}$$

(No other method will be accepted)

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[6] 7. Let T be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that is defined by $T(\tilde{x}) = A\tilde{x}$ where $A = \begin{pmatrix} 3 & 2 & 0 \\ 2 & 5 & 2 \\ 0 & 2 & 3 \end{pmatrix}$. Find all eigenvalues of T.

(DO NOT SOLVE FOR THE EIGENVECTORS)

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[9] 8. Let T be the transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that is defined by $T(\tilde{x}) = A\tilde{x}$ where $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & -2 & 1 \end{pmatrix}$. T has eigenvalues $\lambda = -1, 3$ (One of the eigenvalues has multiplicity 2). Find all eigenvectors associated with each eigenvalue of T.

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