**THE UNIVERSITY OF MANITOBA**

DATE: February 8, 2010 TERM TEST 1
COURSE NO: MATH 1210 TITLE PAGE
EXAMINATION: Techniq. of Classical & Linear Alg. TIME: 1 hour

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NAME: (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

**Please indicate your instructor by placing a check mark in the appropriate box below.**

☐ MWF 9:30 - 10:20 S. Garba. ☐ MWF 1:30 - 2:20 T. Mohammed.

**INSTRUCTIONS TO STUDENTS:**

This is a 1 hour exam. **Please show your work clearly.**

**No texts, notes, or other aids are permitted.** No calculators, cell phones, electronic translators, or electronic devices of any other kind are permitted.

This exam has a title page, 5 pages of questions, and 2 blank pages for rough work. Please check that you have all the pages. **Do not remove the blank pages.**
**Answer all questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

| Question | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 13 | |
| 3 | 13 | |
| 4 | 14 | |
| Total: | 50 | |

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1. Consider the complex number

$$z = \left(\overline{1 - \frac{1}{1+2i}}\right)\left(\frac{4+2i}{5}\right)$$

[5] (a) Express $z$ in Cartesian form, simplify your answer as far as possible.

[5] (b) Express $z$ in exponential form, indication clearly its modulus and the principle value of its argument.

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[13] 2. (a) For $n$ any positive integer use the Principle of Mathematical Induction to prove that

$$\sum_{\ell=n}^{2n} (\ell + 1) = \frac{(n+1)(3n+2)}{2}.$$

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Recall part (a) for any $n \geq 1$;

$$\sum_{\ell=n}^{2n} (\ell + 1) = \frac{(n+1)(3n+2)}{2}.$$

(b) Now use the identity

$$\sum_{j=1}^{m} j = \frac{m\,(m+1)}{2}$$

to show that part (a) is true.

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3. Consider the equation

$$z^3\, i = -8 + 8i$$

[9] (a) Find all solutions to the given equation written in polar form or cartesian form .

[4] (b) Plot the three roots of the above equation on the complex plan, clearly indicating the modulus and the principle value of the argument of each of them.

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4. Consider the real polynomial

$$P(x) = x^4 + x^3 + 3\,x^2 + 9\,x - 54.$$

[3] (a) Find the remainder when $P(x)$ is divided by $(x + 3i)$.

[3] (b) Use Decartes' rule of signs to determine the maximum number of negative real zeros of $P(x) = 0$.

[2] (c) Use the rational roots theorem to list all the possible rational roots of $P(x) = 0$.

[6] (d) Find all zeros, then express $P(x)$ as a product of linear factors only.
(You can use the next page for continuing solving this question.)

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———-Extra Space For Question 4———

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## SCRAP PAPER