Mathematics 1210 Assignment 1 Winter 2012

Hand in on January 20 as instructed by your section lecturer. See the "Template for Induction" on the course webpage for induction questions. Use the HONESTY DECLARATION (see link on course web page) as a cover page for all of your assignments. It is recommended that you write only on one side and attach pages with a single staple in the top left corner, without plastic covers, paper clips or other fasteners.

1. Use mathematical induction to prove that, for all $n \geq 1$,

$$\sum_{i=1}^{n} (2i-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$$

2. Use mathematical induction to prove that, for all $n \geq 1$,

$$1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots + \frac{1}{4^{2n}} = \frac{4^{2n+1} - 1}{3 \cdot 4^{2n}}.$$

3. (a) Find the value of $\displaystyle\sum_{i=1}^{n} \frac{2}{i(i+2)}$ (HINT: Each term is a difference of simple fractions, giving a sum that "telescopes".)

   (b) Use mathematical induction to prove that your answer to part (a) is correct.

4. Use mathematical induction to prove that, for any $n \geq 0$, the polynomial $x^{2n+1} + 1$ is divisible by $x + 1$.

5. Use mathematical induction to prove that, for all $n \geq 1$,

$$2^{n+1} > n^2.$$

   (HINT: You may find it helpful to take $n = 3$ as a base case.)

6. (a) Write $\displaystyle\sum_{k=1}^{m} (2+3k)(4-k)$ as a sum whose terms are themselves sums of the form $c\sum k^a$, where c is some number and a may be any of $0, 1$ or 2.

   (b) Use the formulas $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ to find the value of the sum in part (a).

   (c) Rewrite the sum $\displaystyle\sum_{j=12}^{111} 2^{i-6}(i+9)$ (i.e., by a change of index) so that the resulting initial and terminal index values are 1 and 100.