Math 1210 Assignment 2 Winter 2012

Due Wed Feb 8; hand in as instructed in class. Same instructions as in Assignment #1. Note: one staple in the top left corner is required (the pile is too big for the marker to deal with loose sheets!).

1. Express in the forms required. Unless otherwise instructed, in polar or exponential forms provide the *principal argument*.
   (a) $\dfrac{2+3i}{3-2i} + \left(\overline{1-2i} + (2+i)^3\right)^2$ in Cartesian form.
   (b) $(\sqrt{27} - 3i)^5$ in polar and exponential form;
   (c) $\sqrt{18}\left(\cos\dfrac{19\pi}{4} + i\sin\dfrac{19\pi}{4}\right)$ in Cartesian and exponential form;
   (d) $7e^{-\frac{11\pi}{3}i}$ in Cartesian and polar form.

2. Find all zeros (in $\mathbb{C}$) of the polynomial $f(x) = x^{10} - 64x^5 + 1024$, and their multiplicity (HINT: the polynomial can be factored over $\mathbb{Z}$.) Give your answers in exponential form.

3. Factor the polynomial $g(x) = x^5 + x^4 + 4x^2 + 7x + 3$ over $\mathbb{R}$ into a product of irreducible linear and quadratic factors. (HINT: first find all rational roots.)

4. (a) Use long division to find the quotient and remainder when $x^5 - 3x^4 + 2x^2 - x + 7$ is divided by $x^2 + x + 1$. Express your answer in the form (polynomial) = (polynomial)$\cdot$(quotient)+(remainder).
   (b) Use the remainder theorem to find the remainder when $f(x) = (2+i)x^4 - 2ix^3 + (2-i)x + 2$ is divided by $(1+i)x + 2$.
   (c) For which value(s) of d, if any, is the polynomial $x - 4$ a factor of the polynomial $g(x) = x^3 - 3x^2 + 5x - d$?

5. (a) Find a 6th degree polynomial with *real* coefficients, for which $1+i$ is a zero, $2-i$ is a zero, and -2 is a zero of multiplicity two.
   (b) How many positive zeros does $p(x) = x^7 - x^5 + 4x^4 - x^3 + x^2 + x + 1$ have? How many negative zeros?
   (c) Assuming only that $a, b, c, d, e \in \{0, \pm1, \pm2, \pm3, \pm4, \pm5\}$, use results from Section 2.2.2 to list all possible rational roots of $6x^5 + ax^4 + bx^3 + cx^2 + dx + 5 = 0$.

6. In each case your response should by justified by an appropriate reference (by number) to a Theorem in Section 2.2.1 or 2.2.2.
   (a) Suppose $a, b, c, d, e > 0$, and all the zeros of the polynomial
   $$q(x) = x^5 - ax^4 + bx^3 - cx^2 - dx - e.$$
   are real. How many of them (counting multiplicity) are positive and how many are negative?
   (b) If r has multiplicity 2 as a zero of polynomial $f(x)$ and multiplicity 3 as a zero of polynomial $g(x)$. Explain why it is also a zero of $h(x) = f^2(x) + g^2(x)$. Can we determine it's multiplicity? If so what is the multiplicity? If not, why not?