Math 1210 Assignment 3 Winter 2012

Due Friday March 9; hand in as instructed in class. Same instructions as in Assignment #1. Note: one staple in the top left corner is required. This problem set reprises old material on vectors and geometry because this was poorly done on the midterm.

1. Given points $A(8, 1, -2)$, $B(7, -1, -9)$ and $C(6, -1, -5)$

   (a) Point D is the midpoint of the line segment $\overline{AB}$. Find the coordinates of D.

   (b) Find the coordinates of point E, which is $\frac{1}{3}$ of the way from A to B on segment $\overline{AB}$.

   (c) Find the lengths of all three sides of triangle $\triangle ABC$. What special kind of triangle is this?

2. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be unit vectors. Using everything we know about vectors, simplify the following vector expression as far as you can:

$$((\mathbf{u} \times \mathbf{v}) + \mathbf{u}) \cdot (\mathbf{u} + \mathbf{v} + 6\mathbf{w}) - (\mathbf{v} + 2\mathbf{w}) \cdot (3\mathbf{u} + \mathbf{v})$$

3. Lines and planes:

   (a) Express the line obtained as the intersection of planes $x + 2y - 5z = 10$ and $2x + y - 3z = 20$, in parametric form.

   (b) Find an equation in standard form for the plane perpendicular to the line $\mathbf{v} = [1, 3, 5] + t[0, 1, 2]$ and passing through the point $P_0(1, 1, 1)$

   (c) At which point does line $\mathbf{v} = [3, 2, 1] + t[2, 2, -1]$ meet the plane $x - 2y + 3z = 1$?

4. For the system of equations

$$\begin{array}{rcrcrcrcr} & & v & - & x & - & y & = & 2 \\ -u & + & v & - & 2x & - & 3y & = & 3 \\ u & + & 2v & - & x & & & = & 3 \end{array}$$

   (a) Express the system as a single matrix equation;

   (b) Represent the system in the form of an augmented matrix;

   (c) Put the system in REF using elementary row operations (annotate every step as shown in class);

   (d) Solve the system.

5. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ a & b & c \end{pmatrix}$, where a, b, c are arbitrary.

(a) Which of the following can be obtained from A with a *single* Elementary Row Operation? (Answers only)

$$B_1 = A; \quad B_2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2a+1 & 2b+2 & 2c+3 \end{pmatrix}; \quad B_3 = \begin{pmatrix} -1 & -3 & -5 \\ 1 & 2 & 3 \\ a & b & c \end{pmatrix}$$

(b) Which, if any, of the following can be obtained from A with a sequence of elementary row operations? (Answers only)

$$C_1 = -A; \quad C_2 = \begin{pmatrix} 1+a & 1+b & 1+c \\ 2 & 3 & 4 \\ a-1 & b-2 & c-3 \end{pmatrix}; \quad C_3 = \begin{pmatrix} 1+a & 2+b & 3+c \\ 1+a & 1+b & 1+c \\ 2 & 3 & 4 \end{pmatrix}$$

(c) Show how rows 2 and 3 of the matrix A can be interchanged using a sequence of row operations of *only* the second and third types (multiply a row by a nonzero constant or add a multiple of one row to another row).