Math 1210 Assignment 4 Winter 2012
(UPDATED VERSION!)

Due Friday March 23; hand in as instructed in class. Same instructions as in Assignment #1. One staple in the top left corner is required.

1. Suppose A and B are square 2×2 matrices and $\det(B) \neq 0$. One of the following statements is true and the other is false (in general). Which is which? Justify the true one by mentioning two facts from the course; provide a counterexample for the other one.

   (a) $\det(AB^{-1}) = \frac{\det(A)}{\det(B)}$;

   (b) $\det(A - B) = \det(A) - \det(B)$.

2. Use a series of cofactor expansions along strategically chosen rows/columns to obtain determinant $|A|$, and use a series of elementary row and/or column operations to obtain $|B|$, where:

$$A = \begin{vmatrix} 1 & 2 & 1 & 2 \\ -1 & 0 & 2 & 0 \\ 1 & 3 & 0 & 0 \\ 3 & 0 & -1 & 0 \end{vmatrix} \quad ; \qquad B = \begin{vmatrix} 1 & 2 & 3 & 4 \\ -4 & 3 & -2 & 1 \\ 3 & 1 & 2 & 4 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

3. Solve the following system by following the steps of Gauss-Jordan elimination *exactly*:

$$\begin{array}{rcrcrcr} x_1 & - & x_2 & + & x_3 & = & -2 \\ -x_1 & + & x_2 & + & 3x_3 & = & -1 \\ 3x_1 & - & 3x_2 & + & 2x_3 & = & -10 \end{array}$$

4. Solve the following system using Cramer's Rule:

$$\begin{array}{rcrcrcr} & & -x_2 & + & 2x_3 & = & 2 \\ x_1 & & & + & 2x_3 & = & 4 \\ 2x_1 & + & x_2 & & & = & 6 \end{array}$$

5. Find all values of x such that the following determinant is zero:

$$\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix}$$

6. Let $\mathbf{x}_1$ be a solution to the non-homogeneous system whose matrix equation is $A\mathbf{x} = \mathbf{b}$, and let $\mathbf{x}_2$ be a solution to the corresponding homogeneous system $A\mathbf{x} = \mathbf{0}$. Prove that, for any $c \in \mathbb{R}$, $\mathbf{x}_1 + c\mathbf{x}_2$ is a solution to the first system, $A\mathbf{x} = \mathbf{b}$.

7. Find a real (i.e., all entries real numbers) 2×2 matrix A that satisfies $A^2 = -I$. Use the properties of determinants to show that there are no real 2011×2011 matrices A which satisfy this equation.