Math 1210 Assignment 5 Winter 2012

Due Monday April 2; hand in as instructed in class.

Same instructions always. One staple in the top left corner is required.

1. (a) Express $\mathbf{x} = [5, 0, 3, 3]$ as a linear combination of vectors $\mathbf{u} = [1, 1, -1, 2]$, $\mathbf{v} = [3, 2, 1, 4]$, $\mathbf{w} = [2, 0, 2, 1]$.

   (b) Show that $\mathbf{y} = [1, 1, 1, 1]$ is not a linear combination of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ (from in part 1a).

   (c) Show that every vector in $\mathbb{R}^3$ is a linear combination of vectors $\mathbf{a} = [-1, 1, 1]$, $\mathbf{b} = [1, -1, 1]$ and $\mathbf{c} = [1, 1, -1]$.

2. (a) A 4×4 matrix A has determinant -3. What is the determinant of $\text{adj}A$?

   (b) Find the adjoint of matrix $B = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix}$.

   (c) Find $|B|$; use this and your answer from 2b to find B^{-1}.

   (d) Use your answer from 2c to solve the system

$$\begin{array}{rcrcrcr} x_1 & & & + & 2x_3 & = & 0 \\ 2x_1 & + & 3x_2 & + & x_3 & = & 3 \\ & & x_2 & + & x_3 & = & -3 \end{array}$$

3. Use the *Matrix Inversion Algorithm* (also called the *Direct Method* in the text) to find M^{-1}, where $M = \begin{pmatrix} 4 & 2 & 3 \\ 3 & 3 & 2 \\ 3 & 2 & 2 \end{pmatrix}$.

4. Show that one of the following sets of vectors is dependent, and that the other one is independent.

$$S_1 = \{[1, 1, 0, 0], [1, 0, 1, 0], [0, 1, 0, 1], [0, 0, 1, 1]\}$$

$$S_2 = \{[0, 1, 1, 1], [1, 0, 1, 1], [1, 1, 0, 1], [1, 1, 1, 0]\}$$

5. Let $\mathbf{u}, \mathbf{v}, \mathbf{x}, \mathbf{y}$, be vectors and suppose each of $\mathbf{x}, \mathbf{y}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

   Prove that, if $\mathbf{w}$ is a linear combination of $\mathbf{x}$ and $\mathbf{y}$, then $\mathbf{w}$ is also a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

6. (a) $S : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w}$ and $\mathbf{x}$ are vectors such that

$$S(\mathbf{u}) = [1, 3, -2],\ S(\mathbf{v}) = [2, 2, 0], \text{ and } S(\mathbf{w}) = [-1, 1, 1].$$

   If $\mathbf{x} = \mathbf{u} + 2\mathbf{v} + 3\mathbf{w}$, find $S(\mathbf{x})$.

(b) Suppose $T : \mathbb{R}^4 \to \mathbb{R}^4$ is a linear transformation such that

$$T[1,1,1,1] = [1,2,3,4],\ T[0,1,1,1] = [2,2,0,2],$$

$$T[0,0,1,1] = [2,-1,3,2], \text{ and } T[0,0,0,1] = [0,0,0,0].$$

i. Find the matrix of T.

ii. Use the matrix of T to find $T[3,2,1,1]$.