UNIVERSITY OF MANITOBA

DATE: Feb. 15, 2012, 5:30 PM — MIDTERM

TITLE PAGE

COURSE: MATH 1210 — TIME: 2 hours

EXAMINATION: Classical and Linear Algebra — EXAMINER: Craigen/Klurman

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FAMILY NAME: (Print in ink) ____________________

GIVEN NAME(S): (Print in ink) ____________________

STUDENT NUMBER: __________

SIGNATURE: (in ink) ____________________

(I understand that cheating is a serious offense)

Please indicate your instructor by placing a check mark in the appropriate box below.

☐ MWF 9:30–10:20 O. Klurman ☐ MWF 1:30–1:20 R. Craigen

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 7 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left margin. The total value of all questions is 90 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

For full credit, answers in polar or exponential form must use the *principal argument*.

Question	Points	Score
1	20	
2	8	
3	10	
4	8	
5	8	
6	12	
7	12	
8	12	
Total:	90	

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[20] 1. SHORT ANSWER QUESTIONS

(a) Express the following sum in sigma notation:

$$1 - 5 + 9 - 13 + \cdots - 77 + 81.$$

(b) Simplify the following complex number. Give your answer in Cartesian form.

$$\frac{25i}{3+4i} - \overline{(3-2i)}$$

(c) Put the complex number from part b into polar form.

(d) State the Rational Roots Theorem.

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[8] 2. The following formulas are known from class:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

Use these formulas and the properties of summation to obtain the value of the following sum:

$$\sum_{s=11}^{100} \left(s(s+1)^2 + 1\right) .$$

DO NOT simplify your answer (but eliminate sigma notation).

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[10] 3. Use mathematical induction to show that for all integers $n \geq 1$,

$$3 + 7 + 11 + \cdots + (8n - 1) = 2n(4n + 1).$$

[8] 4. Solve the following equation:

$$x^6 - x^3 + 1 = 0$$

Express your answer as a set of numbers in exponential form.

[8] 5. Find all values of k such that $x - \frac{1}{2}(\sqrt{3} + i)$ is a factor of the polynomial

$$P(x) = x^{24} - 2kx^6 + k^2.$$

(HINT: The Remainder Theorem is helpful. So is working in exponential form.)

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[12] 6. Let $p(x) = 3x^4 + 2x^3 + x^2 + 3x + 5$.
(In this question do not attempt to factor $p(x)$ or to solve $p(x) = 0$)

(a) Use Descartes' Rule of Signs to say what are the possible numbers of positive and negative roots of $p(x) = 0$.

(b) According to the Bounds Theorem, what is the largest absolute value a zero of $p(x)$ may have?

(c) According to the Rational Roots Theorem, what are the possible *rational* zeros of $p(x)$?

(d) Based on the above information, show the smallest set of numbers that would have to be examined, to find all the rational roots of $p(x) = 0$?

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[12] 7. Given vectors $\mathbf{u} = [1, 2, -2]$, $\mathbf{v} = [0, 1, 3]$ and $\mathbf{w} = [2, 2, 1]$

(a) Evaluate $2(3\mathbf{u} + \mathbf{w}) - || \mathbf{u} || (\mathbf{u} + \mathbf{v})$.

(b) Find $\cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

(c) Find a vector that is orthogonal to both $\mathbf{u}$ and $\mathbf{w}$.

(d) Show that the lines
$L_1 : 2\mathbf{u} + s\mathbf{v}$ $(s \in \mathbb{R})$ and
$L_2 : 3\mathbf{v} + t\mathbf{u}$ $(t \in \mathbb{R})$
intersect in a point by finding their point of intersection.

[12] 8. Given the three points $A(1,2,3)$, $B(5,2,-1)$ and $C(3,4,5)$ in $\mathbb{R}^3$, find:

(a) An equation for the plane P containing A, B and C, in point-normal form.

(b) An equation for plane P, in standard form.

(c) A vector equation for the line perpendicular to P and passing through C.

(d) Parametric (scalar) equations for the line through A and B.

(e) Show that $\triangle ABC$ is a right triangle.