MATH 1210 Assignment 1 Winter 2013 Due date: January 25

1. Use mathematical induction on positive integer n to prove each of the following:

(a) $1(3)+2(3^2)+3(3^3)+\cdots+n(3^n)=\frac{1}{4}\left[\,(2n-1)3^{n+1}+3\,\right]$, **for** $n\geq 1$ **;**

(b) $1^2+2^2+3^2+\cdots+(3n)^2=\frac{1}{2}\left[n(3n+1)(6n+1)\right]$, **for** $n\geq 1$ **;**

(c) $5^{2n-1}+1$ **is divisible by** 6 **for** $n\geq 1$ **.**

2. Let a be a real number, use mathematical induction on positive integer $n\geq 1$ to prove that

$$a^{n+1}-1=(a-1)(a^n+a^{n-1}+\cdots+a+1)$$

Do not use any other method.

3. First write sigma form of $(13)^2+(25)^2+(37)^2+\cdots+(24n+1)^2$ and then use identities $\sum_{k=1}^{m}k=\frac{1}{2}\left[m(m+1)\right]$ and $\sum_{k=1}^{m}k^2=\frac{1}{6}\left[m(m+1)(2m+1)\right]$ to prove that

$$(13)^2+(25)^2+(37)^2+\cdots+(24n+1)^2=2n\left(192n^2+168n+37\right).$$

4. Express the sum in sigma notation with summation index starting from 1.

(a) $2+\frac{3}{\sqrt{2}}+\frac{4}{\sqrt{3}}+\frac{5}{2}+\cdots+\frac{101}{10}$ **;**

(b) $2+\frac{1}{\sqrt{2}}+\frac{2}{3\sqrt{3}}+\frac{1}{4}+\frac{2}{5\sqrt{5}}+\cdots+\frac{1}{10\sqrt{20}}$ **;**

(c) $1-\frac{2}{9}+\frac{6}{25}-\frac{24}{49}+\frac{120}{81}-\cdots$ **.**

5. Use formulas $\sum_{k=1}^{m}k=\frac{1}{2}\left[m(m+1)\right]$, $\sum_{k=1}^{m}k^2=\frac{1}{6}\left[m(m+1)(2m+1)\right]$ and $\sum_{k=1}^{m}k^3=\frac{1}{4}\left[m^2(m+1)^2\right]$ to evaluate each of the following sum :

(a) $\sum_{\ell=31}^{41}\left[\frac{1}{22}(\ell-32)^2-\frac{2}{11}\right]$ **;**

(b) $\sum_{j=-11}^{88}\left[(j+12)^3+(j+12)^2-j-12\right]$ **.**

6. Prove that $\sum_{\ell=n}^{\ell=2n}(\ell+1)=\frac{1}{2}\left[(n+1)(3n+2)\right]$ with each of the following two methods:

(a) **By mathematical induction on positive integer $n\geq 1$;**

(b) **By using the identity $\sum_{k=1}^{m}k=\frac{1}{2}\left[m(m+1)\right]$.**

7. For each of the sums $\sum_{n=4}^{203}\frac{9^{n-3}+5^{n-1}}{(n-3)!}$ and $\sum_{n=0}^{199}\frac{3^{2n+2}+5^{n+1}}{(n+1)!}$, change the summation index to 1 and then use it to write $\sum_{n=4}^{203}\frac{9^{n-3}+5^{n-1}}{(n-3)!}-\sum_{n=0}^{199}\frac{3^{2n+2}+5^{n+1}}{(n+1)!}$ as one sum. Simplify as much as possible.