MATH 1210 Assignment 2

Attempt all questions and show all your work. Due Fri Feb 15.

Assignments must include a signed honesty declaration and assignments that do not do so will not be marked.

1. Let z be any complex number. Prove by mathematical induction that for any $n \geq 1$, $(\overline{z})^n = \overline{z^n}$.

2. Let $z = -2+7i$, $w = 3+4i$. Simplify $\dfrac{z^2 - w}{z+w}$ and express in Cartesian form.

3. (a) Find a simplified cartesian form of $z = \left(\dfrac{1}{4} + \dfrac{\sqrt{3}}{4}i\right)^{14}$.

   (b) Express $\left(10e^{(-\pi/6)i}\right)^5$ in Cartesian and polar form.

   (c) Find, in simplified form, the complex number z that satisfies the equation

$$\overline{(5+4i)^2} + (1+4i^3)z = -4i.$$

4. Find all the complex 5th roots of -32. Express your answers in Cartesian form.

5. Find all complex numbers z such that $z^4 + 100z^2 + 10000 = 0$. Write your answers in exponential form.

6. (a) Use long division to find the quotient and remainder when

$$x^5 - 5x^4 - 9x^3 + 49x^2 - 8x - 60$$

   is divided by $x - 3$. Express the result as an equation of the form

$$(\text{polynomial}) = (\text{polynomial})(\text{quotient}) + (\text{remainder}).$$

   (b) Use the remainder theorem to find the remainder when

$$f(x) = (2+i)x^4 + 3x^3 + (2-i)x + 1$$

   is divided by $ix - 2$ (do not perform long division).

   (c) Find all values of d so that the polynomial $2x - 3$ a factor of $g(x) = 6x^3 - 49x^2 + dx + 21$.

   (d) Find all values of h and k such that $(x-2)$ and $(x+1)$ are factors of

$$f(x) = 3x^4 + hx^3 - 15x^2 - 18x + k.$$

7. (a) Factor the polynomial $p(x) = 2x^4 - 9x^3 + 33x^2 - 56x + 30$ completely.

   (b) You are given that $2+i$ is a zero of $p(x) = x^4 + 6x^3 - 10x^2 - 50x + 125$. Write $p(x)$ as a product of linear factors. What are the roots of the equation $p(x) = 0$?

8. Let $f(x)$ and $g(x)$ be two polynomials. If 2 is a zero of $f(x)$ with multiplicity 6, and 2 is also a zero of $g(x)$ with multiplicity 4, must it be a zero of $h(x) = f(x)g(x)$? If yes, what is it's multiplicity? If not, why not? Justify your answer.

9. (a) Let $p(x) = 6x^5 + 2x^4 + 3x^3 - 11x^2 + 10x - 10$. Does $p(x)$ contain any zeros in the interval $(2,4)$? Again, if yes, determine how many.

   (b) Let $q(x) = 5x^4 - 14x^3 + 5x - 9$. Determine with justification (WITHOUT finding any zeros of $q(x)$) whether $q(x)$ has any zeros in the interval $[4,10]$, and if so, how many.