MATH 1210 Winter 2013 Assignment 3

Attempt all questions and show all your work. The assignment is due Friday March 22nd.

1. Let $A = \begin{bmatrix} 0 & 2 \\ 3 & 5 \end{bmatrix}$. Show by induction that for all integers $n \geq 1$,

$$A^n = \frac{1}{7}\begin{bmatrix} 6(-1)^n + 6^n & -2(-1)^n + 2 \cdot 6^n \\ -3(-1)^n + 3 \cdot 6^n & (-1)^n + 6 \cdot 6^n \end{bmatrix}$$

2. Let $\mathbf{u} = \langle 2, 3, 4 \rangle$ and $\mathbf{v} = \langle 1, 2, 3 \rangle$.

   (a) Find $\mathbf{u} \times \mathbf{v} - 2\hat{\mathbf{u}}$ where $\hat{\mathbf{u}}$ is the unit vector in the direction of $\mathbf{u}$.

   (b) Find the cosine of the angle between $2\mathbf{u} - 3\mathbf{v}$ and $3\mathbf{u} - 2\mathbf{v}$.

3. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in 3 dimensions.

   (a) Show that $|\mathbf{u}|^2 = \mathbf{u} \cdot \mathbf{u}$.

   (b) Show that
   $$|\mathbf{u} + \mathbf{v}|^2 + |\mathbf{u} - \mathbf{v}|^2 = 2\,|\mathbf{u}|^2 + 2\,|\mathbf{v}|^2$$

4. Let l_1 and l_2 be the lines
   $$l_1 : x - 2y + 2z = 5, \quad 3x + y + z = 3$$
   and
   $$l_2 : \frac{x-1}{3} = \frac{2y+2}{4} = \frac{1-z}{2}.$$

   (a) Find parametric equations for l_1.

   (b) Find the point of intersection of the two lines

   (c) Find the point of intersection of the line l_2 and the plane $2x + y + z = 14$.

   (d) Find the equation of the plane which contain both lines.

5. Use Gauss-Jordan elimination to solve the following systems of linear equations.

   (a)
   $$\begin{aligned} 2x + 3y - z &= 6 \\ x - y + 3z &= -4 \\ 3x + 7y - 5z &= 16 \end{aligned}$$

   (b)
   $$\begin{aligned} 3x + y - 3z &= 6 \\ x + 2y + 3z &= 4 \\ 3x - 4y - 15z &= 2 \end{aligned}$$

(c)

$$\begin{aligned} 2x + y - z + 2w &= 6 \\ 3x - 2y + 3z - w &= -4 \\ 7y - 9z + 8w &= 26 \end{aligned}$$

6. Find the value(s) of a such that the following system

$$\begin{aligned} x + 2y + 5z &= 1 \\ 5y + 13z &= 11 \\ -5y + (a^2 - 29)z &= a - 15 \end{aligned}$$

has:

(a) no solutions.

(b) exactly one solution.

(c) infinitely many solutions.

7. Solve the following non-linear system of equations for x, y, z.

$$\begin{aligned} \frac{1}{x} + \frac{2}{y} - \frac{4}{z} &= 1 \\ \frac{2}{x} + \frac{3}{y} + \frac{8}{z} &= \frac{11}{2} \\ -\frac{1}{x} + \frac{9}{y} + \frac{10}{z} &= 6 \end{aligned}$$

8. Find basic solutions of a homogeneous system whose augmented matrix is

$$\left[\begin{array}{cccccc|c} 1 & 2 & 0 & 0 & 3 & -2 & 0 \\ -2 & -4 & 1 & 0 & -5 & 0 & 0 \\ -1 & -2 & 0 & 1 & -2 & 1 & 0 \end{array}\right]$$

9. Let $U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$ and $V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ be solutions to the homogeneous system of linear equations $AX = \mathbf{0}$. Show that any linear combination of U and V is also a solution.