MATH 1210 Assignment 4 Winter 2013 **Due date: April 5**

1. Determine whether each of the following matrices is invertible. If yes find the inverse and if no explain why.

$$(a)\quad A=\begin{pmatrix}-1&2&-3\\2&1&0\\4&-2&5\end{pmatrix}\qquad (b)\quad B=\begin{pmatrix}-40&16&9\\13&-5&-3\\5&-2&-1\end{pmatrix}\qquad (c)\quad C=\begin{pmatrix}3&1&2\\1&-2&-4\\-5&3&6\end{pmatrix}$$

2. Use properties of determinant to prove that

$$\begin{vmatrix}1&1&1\\a&b&c\\(a+1)^2&(b+1)^2&(c+1)^2\end{vmatrix}=(b-a)(c-a)(c-b)\,.$$

3. Find the value of x such that
$$\begin{vmatrix}x&2&1\\-1&0&1\\0&3&x\end{vmatrix}=\begin{vmatrix}0&x&-1\\2&3&4\\0&1&-2\end{vmatrix}.$$

4. Let A, B and C be 5×5 matrices such that $det(A)=3$, $det(B)=-2$ and $det(C)=10$. Evaluate each of the following:
   (a) $det(AB^TC)$,
   (b) $det(A^2\,(B^T)^{-1})$,
   (c) $det(A^{-1}DB^{-3}D^{-1})$, (where D is another 5×5 matrix).

5. Find all values of x for which the matrix $A=\begin{pmatrix}x&1-x&3\\1&x&-1\\2&1&1\end{pmatrix}$ is singular (that is not invertible).

6. Let $A=\begin{pmatrix}2&4&1&1\\2&4&1&2\\2&4&2&3\\2&5&0&1\end{pmatrix}$. Find $|A|$ by using properties of determinants to create an upper triangular matrix. For each elementary operation you use, explain what effect it has on determinant.

7. Let $A=\begin{pmatrix}1&1&2\\0&3&-1\\2&1&4\end{pmatrix}$. First find A^{-1} then find all solutions of each of the following systems:
   (a) $A\mathbf{x}=\begin{pmatrix}2\\-1\\3\end{pmatrix}$, (b) $(-3A)\mathbf{x}=\begin{pmatrix}-3\\0\\6\end{pmatrix}$, (c) $A^{-1}\mathbf{x}=\begin{pmatrix}2\\-1\\1\end{pmatrix}$, (d) $A^T\mathbf{x}=\begin{pmatrix}3\\1\\0\end{pmatrix}$

8. Let $A=\begin{pmatrix}0&0&0&3\\2&0&0&0\\0&2&5&-2\\0&0&-3&0\end{pmatrix}$.
   (a) Find $det(A)$.
   (b) If $adj(A)=\begin{pmatrix}0&18&0&0\\12&0&b&30\\0&0&0&-12\\a&0&0&0\end{pmatrix}$, find the values of a and b.
   (c) Find A^{-1}.

9. Let $\mathbf{u} = \langle 1, 3, -1 \rangle$, $\mathbf{v} = \langle 0, 2, 2 \rangle$ and $\mathbf{w} = \langle -1, 1, 4 \rangle$. Is $\mathbf{r} = \langle 3, 4, 6 \rangle$ a linear combination of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$? If yes write $\mathbf{r}$ in terms of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$; if no explain why?

10. For each of the following parts determine if the given vectors are linearly dependent or linearly independent. Show your work.
    (a) $\mathbf{u} = \langle 3, 1, 3 \rangle$, $\mathbf{v} = \langle 1, -4, 14 \rangle$, $\mathbf{w} = \langle 4, 5, -7 \rangle$,
    (b) $\mathbf{u} = \langle 4, 7 \rangle$, $\mathbf{v} = \langle 8, 11 \rangle$, $\mathbf{w} = \langle 12, 13 \rangle$,
    (c) $\mathbf{u} = \langle 1, 0, 2, 1 \rangle$, $\mathbf{v} = \langle 3, -1, 1, 4 \rangle$, $\mathbf{w} = \langle 4, -1, 3, 0 \rangle$,
    (d) $\mathbf{u} = \langle -\frac{1}{2}, 2, \frac{3}{4}, 4 \rangle$, $\mathbf{v} = \langle 1, -4, -\frac{3}{2}, -8 \rangle$, $\mathbf{w} = \langle 5, -7, 1, 1 \rangle$.

11. Given that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are linearly independent, prove that $2\mathbf{u} + \mathbf{v}$, $3\mathbf{v} - \mathbf{u}$ and $2\mathbf{v} + \mathbf{w}$ are also linearly independent.