UNIVERSITY OF MANITOBA

DATE: February 27, 2013 MIDTERM

TITLE PAGE

EXAMINATION: Techniques of Classical and Linear Algebra TIME: 1 hour

COURSE: MATH 1210 EXAMINER: Borgersen, Harland, Moghaddam

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NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________

SEAT NUMBER: ______________

SIGNATURE: (in ink) ______________________________

(I understand that cheating is a serious offense)

- ❑ A01 9:30–10:20 AM MWF (100 St. Paul) N. Harland
- ❑ A02 1:30–2:20 PM MWF (221 Wallace) G. I. Moghaddam
- ❑ A03 9:30–10:20 AM MWF (136 Art Lab) R. Borgersen

INSTRUCTIONS TO STUDENTS:

This is a 1 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 6 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 70 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	8	
2	6	
3	7	
4	7	
5	15	
6	6	
7	9	
8	12	
Total:	70	

[8] 1. Use mathematical induction on positive integer $n \geq 2$ to prove that

$$(1-\frac{1}{2^2})(1-\frac{1}{3^2})(1-\frac{1}{4^2})\cdots(1-\frac{1}{n^2}) = \frac{n+1}{2n}\,.$$

[6] 2. Use the formula $\displaystyle\sum_{k=1}^{m} k^2 = \frac{1}{6}\,[m(m+1)(2m+1)]$ to evaluate $\displaystyle\sum_{j=-5}^{4} \big[(j+7)^2 - 2(j+6)\big]$.

[7] 3. Find, in simplified form, the Cartesian form of $(\sqrt{6} + \sqrt{2}\,i)^{10}$.

[7] 4. Find, in simplified form, the complex number z that satisfies the equation

$$(1-i)^2(1+i)^2 + \overline{(4-3i)}\, z = 1+4i\,.$$

5. Consider the polynomial equation of $P(x) = 0$ where

$$P(x) = 3x^4 - 18x^3 + 28x^2 + 12x - 20\,.$$

[3] (a) Use Descartes' Rule of Signs to find the possible number of positive and negative roots of $P(x) = 0$.

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[3] (b) What are the possible rational zeros of $P(x) = 0$?

[3] (c) Use Bounds Theorem to eliminate some of the possible zeros you found in part (b).

[6] (d) Given that $3 + i$ is a root of $P(x)$, find all other roots.

[6] 6. Let $A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 3 & 0 & 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 4 & 1 & -1 \end{pmatrix}$. Find a matrix X such that $AB^T - X = I$.

7. For $a > 0$ let $\mathbf{u} = < 2a, -a, a >$, $\mathbf{v} = < 1, 1, 2 >$ and $\mathbf{w} = < 3, 4, 1 >$.

[5] (a) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

[4] (b) Find the value of a for which $\mathbf{u} + \mathbf{v}$ and $\mathbf{w}$ are perpendicular.

8. Consider the point $P(2,3,5)$ and the two planes $\Pi_1 : \ 2x+3y-z=0$ and $\Pi_2 : \ -x+y+z=0$.

[2] (a) Is the point P on the plane Π_1?

[4] (b) Show that the two planes Π_1 and Π_2 are perpendicular.

[6] (c) Find parametric equations of the line through the point P and parallel to the line $2x+3y-z=0$, $-x+y+z=0$.