UNIVERSITY OF MANITOBA

DATE: February 25, 2014 MIDTERM
TITLE PAGE

EXAMINATION: Techniques of Classical and Linear Algebra TIME: 75 minutes
COURSE: MATH 1210 EXAMINERS: Chipalkatti, Kucera, Moghaddam

FAMILY NAME: (Print in ink) _____

GIVEN NAME: (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense.)

Please place a check mark (✓) in the box next to your section.

❑	A01	9:30–10:20 AM	MWF (110 E2 EITC)	G. I. Moghaddam
❑	A02	1:30–2:20 PM	MWF (100 St. Paul's)	J. Chipalkatti
❑	A03	9:30–10:20 AM	MWF (100 St. Paul's)	T. Kucera

INSTRUCTIONS TO STUDENTS:

This is a 75 minutes exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 7 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin next to the statement of the question. The total value of all questions is 68 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	8	
2	10	
3	14	
4	4	
5	8	
6	8	
7	8	
8	8	
Total:	68	

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[8] 1. Use mathematical induction to prove that

$$\sum_{j=1}^{n} 3(j+1)(j-2) = n(n^2-7)\,,$$

for all integers $n \geq 1$.

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[10] 2. Find, in simplified form, the Cartesian form of each of the following expressions:

(a) $\dfrac{1-i}{i-\frac{1}{1-i}}$

(b) $(\sqrt{2}-i\sqrt{6})^6$

[14] 3. Consider the polynomial equation of $P(x) = 0$ where

$$P(x) = 3\,x^4 + x^3 - 6\,x^2 - 5\,x - 1$$

(a) Use the Rational Root Theorem to list all possible rational roots of $P(x)$.

(b) Use direct substitution to check whether any of them are in fact roots.

(c) Use your results from (b) to find all the roots of $P(x)$.

[4] 4. Let z denote any complex number. Prove that if $|z| = 1$, then $\overline{z} = \dfrac{1}{z}$.

[8] 5. Consider the polynomial equation of $P(x) = 0$ where

$$P(x) = 3x^7 - 5x^6 - 11x^5 + 8x^3 + 7x^2 - 13x + 9$$

(a) Use the Bounds Theorem to deduce a bound on $|z|$, where z denotes any root of $P(x)$.

(b) Use Descartes' Rule of Signs to find the possible numbers of positive and negative real roots.
(Your are **not** asked to find the roots of $P(x)$.)

[8] 6. Let $A = \begin{pmatrix} 5 & -4 & 3 \\ -1 & 2 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ 1 & 0 \end{pmatrix}$. Evaluate each of the following expressions or explain why it is not defined.

(a) $AA^T - B^2$.

(b) $(2B - B^T)A^T$.

[8] 7. Let $\mathbf{u} = \langle\, 2\,,\, -3\,,\, 5\,\rangle$, $\mathbf{v} = \langle\, 1\,,\, 2\,,\, -3\,\rangle$ and $\mathbf{w} = \langle\, k+4\,,\, k\,,\, 2k\,\rangle$.

(a) Find the value of k for which $\mathbf{u}+\mathbf{v}$ and $\mathbf{w}$ are perpendicular.

(b) Find $(\mathbf{u}+\mathbf{v}) \times (\mathbf{u}-\mathbf{v})$.

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[8] 8. Consider the point $P(4,1,1)$, the plane $\Pi : 3x - y + 2z = -1$, and the line $\ell : \; x = 1 + t\,,\; y = -2 + 2t\,,\; z = t\,.$

(a) Find the point of intersection of the line ℓ and the plane Π.

(b) Find an equation of the plane which passes through the point P and is perpendicular to the line ℓ.

For rough work only; no work on this page will be marked.