UNIVERSITY OF MANITOBA

DATE: April 23, 2015 FINAL EXAMINATION

TITLE PAGE

EXAMINATION: Techniques of Classical and Linear Algebra TIME: 2 hours

COURSE: MATH 1210 EXAMINERS: various

FAMILY NAME (Write in Capital Letters): ______________________

GIVEN NAME (Write in Capital Letters): ______________________

STUDENT NUMBER: ______________________

SIGNATURE: (in ink) ______________________________

(I understand that cheating is a serious offense.)

- ❏ A01 9:30–10:20 AM MWF (207 Buller) Dr. R. Padmanabhan
- ❏ A02 1:30–2:20 PM MWF (206 Human Ecology) Dr. N. Harland
- ❏ A03 9:30–10:20 AM MWF (205 Armes) Dr. J. Chipalkatti

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page and 10 pages of questions. **DO NOT REMOVE ANY PAGES OR THE STAPLE.**

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued. You may use the back of the pages for scrap work, but be **very clear** if you want it marked.

Question	Points	Score
1	9	
2	10	
3	7	
4	9	
5	10	
6	9	
7	6	
8	9	
9	12	
10	7	
11	12	
Total:	100	

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[9] 1. Let A be the 2×2 matrix $\begin{bmatrix} 3 & -1 \\ 4 & -1 \end{bmatrix}$. Use the principle of mathematical induction to prove that

$$A^n = \begin{bmatrix} 2n+1 & -n \\ 4n & 1-2n \end{bmatrix}$$

for all $n \geq 1$.

2. Let $P(x)$ be the polynomial $P(x) = 2x^3 - 5x^2 + 6x - 2$.

[3] (a) Verify that $1 + i$ is a root of the equation $P(x) = 0$.

[7] (b) Find all roots of the equation $P(x) = 0$.

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[7] 3. The following two lines are parallel:

$$L_1: \quad \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-2}{5}, \qquad \text{and } L_2: \quad \frac{x+4}{2} = \frac{y}{-1} = \frac{z+1}{5}.$$

You do not have to show they are parallel. Find the standard equation of the plane containing them.

4. Consider the vectors

$$\mathbf{v} = \langle 1, -1, 4 \rangle, \quad \mathbf{w} = \langle 2, -7, -3 \rangle$$

and let θ denote the smallest angle between them.

[3] (a) Use the dot product of $\mathbf{v}$ and $\mathbf{w}$ to find $\cos\theta$.

[4] (b) Use the cross product of $\mathbf{v}$ and $\mathbf{w}$ to find $\sin\theta$.

[2] (c) Verify that $\cos^2\theta + \sin^2\theta = 1$.

5. Let A and B denote 5×5 matrices such that $\det(A) = -3$ and $\det(B) = 4$. Find the determinant of

[3] (a) BA^{-1}

[4] (b) $(\mathrm{adj}(A))^{-1}$

[3] (c) $\dfrac{1}{2}AB^T$

[9] 6. Consider the vectors

$$\mathbf{u} = \langle 1, 5, -4 \rangle, \quad \mathbf{v} = \langle 2, -4, 2 \rangle, \quad \mathbf{w} = \langle 1, -30, 21 \rangle.$$

Verify that the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are linearly dependent and express $\mathbf{w}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

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[6] 7. Consider the system of equations

$$tx - y = 2, \quad x - ty = 1 + t,$$

in the variables x and y. Find all values of t for which the system is inconsistent.

[9] 8. The matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ -3 & 5 & 9 \end{pmatrix}$$

has determinant of -4 and the adjoint is

$$\mathrm{adj}(A) = \begin{pmatrix} x & -3 & 1 \\ -42 & 18 & y \\ 25 & -11 & 1 \end{pmatrix}.$$

Find x, y, and A^{-1}.

9. For the matrix $A = \begin{pmatrix} 1 & 3 & -1 \\ 3 & 9 & -2 \\ 0 & 1 & 2 \end{pmatrix}$:

[6] (a) Use the direct method, also called the row reduction method, to find A^{-1} if it exists. (No credit will be given to any other method.)

[3] (b) Use the information from part (a) to solve the system

$$\begin{aligned} x + 3y - z &= 1 \\ 3x + 9y - 2z &= 2 \\ y + 2z &= 3. \end{aligned}$$

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[3] (c) Use the information from part (a) to solve the system

$$\begin{aligned} x + 3y &= 1 \\ 3x + 9y + z &= 2 \\ -x - 2y + 2z &= 3. \end{aligned}$$

10. Let T be the transformation

$$T: \quad \begin{array}{ll} v_1' & = v_1 - 3v_2 \\ v_2' & = 2v_1 - 8v_2 \end{array}.$$

[2] (a) Is T linear? Justify your answer.

[1] (b) Find $T\langle 2, 5\rangle$.

[4] (c) Find $\mathbf{v}$ such that $T(\mathbf{v}) = \langle 1, 2\rangle$.

11. Let A be the matrix $\begin{pmatrix} -1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 4 & 5 \end{pmatrix}$.

[6] (a) Find all the eigenvalues of A. (Note: it may be helpful to use that $\lambda = -1$ is an eigenvalue.)

[6] (b) Find two linearly independent eigenvectors corresponding to $\lambda = -1$.