The University of Manitoba

MATH 1210: Techniques of Classical and Linear Algebra
(Winter Term 2015)

Midterm-1
February 24th, 2015

Time: One hour   Total Marks: 50

Last Name (CAPITAL LETTERS ONLY): _____

First Name (CAPITAL LETTERS ONLY): _____

Student Number: _____

Signature: _____

(I acknowledge that cheating is a serious offense.)

Place a check mark (✓) in the box corresponding to your section and instructor.

☐   Dr J. Chipalkatti   M-W-F 9:30 in 205 Armes
☐   Dr N. Harland   M-W-F 1:30 in 206 Human Ecology
☐   Dr R. Padmanabhan   M-W-F 9:30 in 207 Buller

Instructions:

Please ensure that your paper has a total of 5 pages (including this page). Read the questions thoroughly and carefully before attempting them.

You are **not allowed** to use any of the following: calculators, notes, books, dictionaries or electronic communication devices (e.g., cellular phones, pagers or blackberries).

You may use the back pages for scratch work.

	Obtained	Maximum
Q1		8
Q2		6
Q3		7
Q4		6
Q5		12
Q6		5
Q7		6
Total		**50**

Q1. Use the principle of mathematical induction to prove the identity: [8]
$$2 + 5 + 8 + \cdots + (6n - 1) = 6\,n^2 + n \qquad \text{for } n \geqslant 1.$$

Q2. Find the sum $\sum_{k=4}^{9} (k^2 + k)$. You may use the following formulae: [6]
$$\sum_{i=1}^{n} i = \frac{n\,(n + 1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n\,(n + 1)\,(2\,n + 1)}{6}.$$

Q3. Write the following complex number in Cartesian form: [7]
$$\frac{(1+i)^6}{(1-i)^7}.$$

Q4. Find all solutions to the equation [6]
$$z^2 = 5 + 12\,i.$$

Hint: You may assume $z = x + i\,y$, where x, y are real. Now find a set of equations involving x and y.

Q5. Consider the polynomial [12]

$$P(x) = 3\,x^3 - 2\,x^2 + 5\,x + 2.$$

(a) Use the Bounds Theorem to find an upper bound for $|\alpha|$, where α denotes any root of $P(x)$.

(b) Use the Rational Root Theorem to find the list of possible rational roots of $P(x)$.

(c) Use direct substitution to check whether any of these are in fact roots.

(d) Use your results from (c) to find all the roots of $P(x)$.

(e) Verify that all the roots found in (d) satisfy the upper bound in (a).

Q6. Consider the polynomial [5]

$$P(x) = 100\,x^4 + 32\,x^3 - 26\,x^2 + t\,x - 1,$$

where t is a real number. It is given that the roots of $P(x)$ are:

$$0.56, \quad -0.15, \quad -0.26, \quad -0.47.$$

Is t positive, negative or zero? You must give adequate justification for your answer.

Q7. Consider the matrix [6]

$$A = \begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}.$$

Find the matrix

$$A^2 - 3\,A - I_2.$$