MATH 1210 Assignment 1 17R-T2

This assignment is **optional** and does not need to be handed in. Attempt all questions, write out nicely written solutions (showing all your work), and the solutions will be posted on Fri, Feb 3, 2017, at which point you can mark your own work. If you have any questions regarding differences between what you wrote and what the solution key says, please contact your professor. **At least one question from this assignment will be found on Quiz 1.**

1. Show that for all $n \geq 1$, $4^3 + 8^3 + \cdots + (4n)^3 = 16n^2(n+1)^2$.

2. (a) Write the sum $1 + 3 + 5 + \cdots + (4n - 1)$ using sigma notation.

   (b) Use Mathematical Induction to prove that for all $n \geq 1$, the above expression is equal to $(2n)^2$.

3. Use Mathematical Induction to prove that for all $n \geq 1$,

$$n + (n+1) + (n+2) + \cdots + (2n) = \frac{3n(n+1)}{2}.$$

4. Use Mathematical Induction to prove that for all $n \geq 1$,

$$1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^{2n}} = \frac{3}{2}\left(1 - \left(\frac{1}{3}\right)^{2n+1}\right).$$

5. Use Mathematical Induction to prove that for all $n \geq 1$, $3^n > n^2$.

6. Consider the sequence of real numbers defined by the relations $x_0 = 0.5$ and for all $n \geq 1$, $x_n = 0.5x_{n-1}(1 - x_{n-1})$. Prove by induction that for all $n \geq 0$, $x_n \in (0, 1)$.

7. (a) Express the sum $\displaystyle\sum_{k=1}^{m}(2 + 3k)^2$ in terms of three simpler sums in sigma notation by expanding. Do not calculate the value.

   (b) Find the value of the sum

$$\sum_{p=1}^{100}(2 - 10p + 3p^2).$$

   HINT: Make use of the formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \text{and} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

   (c) Rewrite the sum

$$\sum_{r=12}^{122} \frac{r-6}{r+9}$$

   using an index whose initial and terminal values are 1 and 111 (HINT: use a change of variables).