This assignment is **optional** and does not need to be handed in. Attempt all questions, write out nicely written solutions (showing all your work), and the solutions will be posted on Fri, Feb 24, 2017, at which point you can mark your own work. If you have any questions regarding differences between what you wrote and what the solution key says, please contact your professor. **At least one question from this assignment will be found on Quiz 2.**

1. Simplify and express the complex numbers in Cartesian form

   (a) $2e^{i\frac{\pi}{4}} + 3e^{i\frac{-\pi}{4}}$

   (b) $\overline{\left(\frac{(6-2i)^4}{(1+3i)^4}\right)}$

   (c) $\frac{(i-1)^{10}}{(i+1)^{13}}$

   (d) $\left(\frac{i}{e^{i\pi}}\right)^{25}$

2. Simplify and express the complex numbers in polar and exponential forms using the principal value of the argument θ, $\theta \in (-\pi, \pi]$

   (a) $\left(\overline{\sqrt{3}+3i}\right)^2$

   (b) $(-12+i)^3(-12-i)^3$

   (c) $\dfrac{(1-\sqrt{3}i)^{10}}{(1+\sqrt{3}i)^{10}}$

3. Find all solutions of the equation

   $x^6 + x^3 + 1 = 0.$

4. Find all solutions of the equation $z^8 = -1$. Express your answers with the argument between $-\pi$ and π.

5. Find all solutions of the equation $z^4 = i$.

6. Let z_1 and z_2 be 2 complex numbers. Show that

   $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}.$

7. Let z be a complex number. Using mathematical induction prove that

   $\overline{z^n} = \overline{z}^n$, for all $n \geq 1$.

8. Consider the following polynomial $P(x) = x^5 - 2x^4 + 4x^3 + 2x^2 - 5x$.

   (a) Verify that $1+2i$ is a root of $P(x) = 0$.

   (b) Find all the roots of $P(x) = 0$.

   (c) Factor $P(x)$ into the product of real linear and irreducible real quadratic factors.

9. (a) Show that $(x-i)$ and $(x-1)$ are linear factors of

   $x^4 - 2(1+i)x^3 + 4ix^2 + 2(1-i)x - 1 = 0.$

   (b) Factor the polynomial $x^4 - 2(1+i)x^3 + 4ix^2 + 2(1-i)x - 1$ in linear factors.

10. Consider the following polynomial

    $P(x) = x^5 - 11x^4 + 43x^3 - 73x^2 + 56x - 16.$

    (a) Show that $P(x)$ can be rewritten as $P(x) = Q(x)(x-4)$ and $P(x) = T(x)(x-1)$ where $Q(x)$ and $T(x)$ are polynomials in x. Give the degree of $Q(x)$ and $T(x)$.

    (b) Show that 4 is a root of multiplicity 2 of $P(x)$.

    (c) Factor $P(x)$.

11. For each of the following polynomials:

$$P_5(x) = 6x^5 + 7x^4 - 13x^3 - 85x^2 - 50x$$

$$P_9(x) = x^9 + 3x^8 + 3x^7 + 3x^6 + 6x^5 + 6x^4 + 4x^3 + 6x^2 + 6x + 2$$

    (a) Use Descartes' rules of signs to state the number of possible positive and negative zeros of the polynomial;

    (b) use the bounds theorem to find bounds for zeros of the polynomial;

    (c) use the rational root theorem to list all possible rational zeros of the polynomial;

    (d) use this information to find all the zeros of the polynomial.

12. Let

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 5 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} -1 & 5 & 2 \\ 1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 5 & 1 & 3 \end{bmatrix}.$$

    Evaluate each of the following expressions or explain why it is undefined.

    (a) $(2D^T - E)A$

    (b) $(4B)C + 3B$

    (c) $D^T(4E^T) - 4(ED)^T$