MATH 1210 Assignment 4 17R-T2

This assignment is **optional** and does not need to be handed in. Attempt all questions, write out nicely written solutions (showing all your work), and the solutions will be posted on Fri, March 24, 2017, at which point you can mark your own work. If you have any questions regarding differences between what you wrote and what the solution key says, please contact your professor. **At least one question from this assignment will be found on Quiz 4.**

1. Solve the linear system

$$\mathbf{AX} = \mathbf{B}$$

where $\mathbf{B}^T = [0, 0, 0]$ and

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 2 & -3 & -1 & -4 \\ 3 & -5 & -1 & -1 \end{bmatrix}$$

Specify the method used to solve the linear system.

2. Compute the determinant of

(a) $\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 2 & -3 & -1 & -4 \\ 3 & -5 & -1 & -1 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & -3 & 4 \\ 0 & 2 & -2 & 3 \\ 1 & 1 & -4 & -2 \end{bmatrix}$

(c) $\mathbf{A}^5$ when $\mathbf{A}$ is defined as in (b).

(d) $-\mathbf{A}$ when $\mathbf{A}$ is defined as in (b).

(e) $\mathbf{A}^T$ when $\mathbf{A}$ is defined as in (b).

3. Find all values of λ for which $\det(\mathbf{A}) = 0$ for $\mathbf{A} = \begin{bmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{bmatrix}$.

4. Consider the following linear system

$$\begin{aligned} 2x - y + 2z &= 2, \\ x - y - z &= 1, \\ 4x + 2y - z &= 0. \end{aligned}$$

(a) What is the rank of the augmented matrix of the linear system?

(b) What is the rank of the coefficient matrix of the linear system?

(c) By using Gauss-Jordan elimination, solve the linear system.

5. Consider the following linear system

$$\begin{aligned} 2x_1 + 2x_2 - x_3 + x_5 &= 0, \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0, \\ x_1 + x_2 - 2x_3 - x_5 &= 0, \\ x_3 + x_4 + x_5 &= 0. \end{aligned}$$

   (a) By using Gauss-Jordan elimination, solve the linear system.

   (b) Write your solution(s) using the basic solution(s).

   (c) Is the trivial solution a solution of this linear system? Explain your answer.

6. Consider the linear system:

$$\begin{aligned} 2x + 2y - 3z &= -1 \\ 2x + y - z &= -1 \\ 3x - 2y + z &= -4 \end{aligned}$$

   (a) Use elementary row operations (i.e., Gaussian or Gauss-Jordan elimination) to solve it.

   (b) Use Cramer's rule to solve it.