The University of Manitoba

MATH 1210: Techniques of Classical and Linear Algebra (Winter Term 2018)

Midterm Examination
February 28th, 2018

Time: 75 minutes | Total Marks: 50

Last Name (in capital letters): ______________________________

First Name (in capital letters): ______________________________

Student Number: ______________________________

Signature: ______________________________

(I acknowledge that cheating is a serious offense.)

Place a check mark (✓) in the box corresponding to your section and instructor.

- ☐ Jaydeep Chipalkatti M-W-F at 1:30pm in 205 Armes
- ☐ Tyrone Ghaswala M-W-F at 9:30am in EITC E2 110
- ☐ Sergei Tsaturian M-W-F at 9:30am in 100 St. Paul's

Instructions:
Please ensure that your paper has a total of 5 pages (including this page). Read the questions thoroughly and carefully before attempting them. You must **show your work** clearly in order to get any marks for your answers.

You are **not allowed** to use any of the following: calculators, notes, books, dictionaries or electronic communication devices (e.g., cell-phones or pagers).

You may use the back pages for scratch work. However, it will not be marked.

	Obtained	Maximum
Q1		7
Q2		7
Q3		6
Q4		6
Q5		6
Q6		6
Q7		6
Q8		6
Total		**50**

Q1. Use the principle of mathematical induction to prove the identity [7]

$$2+7+12+17+\cdots+(5\,n-3)=\frac{n\,(5\,n-1)}{2},$$

for $n \geqslant 1$.

Q2. Express the complex number $(\overline{2\,e^{i\,\pi/3}})^4$ in Cartesian form. [7]

Q3. Find all solutions to the equation [6]

$$z^3 = 1 + i.$$

Express your solutions in exponential form.

Q4. Consider the polynomial [6]

$$f(x) = x^3 + 4x^2 + kx + 3.$$

It is given that if you divide $f(x)$ by $x + 3$, then the remainder is $k + 1$. Find the value of k.

Q5. Consider the polynomial [6]

$$g(x) = x^3 - t\,x^2 - 1,$$

where t is an integer. Find all values of t for which $g(x)$ has a rational root.

Q6. Let [6]

$$A = \begin{bmatrix} -1 & 2 \\ 4 & 5 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 \\ 0 & -7 \\ 5 & 2 \end{bmatrix}.$$

Find the matrix $A^T B - I_2$.

Q7. Let P be the plane in $\mathbb{R}^3$ defined by the equation $x+2y-z=3$. Find the parametric equations of the line perpendicular to P which passes through the point $(3,0,3)$. [6]

Q8. Consider the vectors $\mathbf{u} = \langle 1, -1, 2 \rangle$, and $\mathbf{v} = \langle 3, 1, -7 \rangle$ in $\mathbb{R}^3$. Find a vector $\mathbf{w}$, other than the zero vector, such that [6]

$$\mathbf{u} \cdot \mathbf{w} = 0, \quad \text{and} \quad \mathbf{v} \cdot \mathbf{w} = 0.$$