## MATH 1210 (WINTER TERM 2019) ASSIGNMENT-2

Please complete the entire assignment, staple it to the Honesty Declaration Form (which should serve as the front page) and submit it to your instructor by February 15th, 2019. Only some of the questions will be marked. Late assignments will not be accepted.

Q1. Recall that the Bounds Theorem (Theorem 3.9 in the textbook) is valid for polynomials with complex coefficients and complex roots. You only have to interpret $|x|$ as the modulus of $x$ throughout. Now consider the degree 3 polynomial

$$
p(x)=(1-i) x^{3}-(8+2 i) x^{2}+(7+13 i) x+(50-20 i) .
$$

It is given to you that its roots are

$$
z=3-i, \quad 2+5 i, \quad-2+i
$$

(You do not have to check this.)
(1) Use the Bounds Theorem to find a real positive number $Q$ such that

$$
|z|<Q
$$

for each of the roots $z$.
(2) Find the modulus of each root, and verify that this inequality is true of each of them.

Q2. Consider the polynomial

$$
f(x)=7 x^{5}-13 x^{4}-2 x^{3}+7 x^{2}-13 x-2 .
$$

(1) Use the Rational Roots Theorem to find the possible rational roots of $f(x)$.
(2) Use direct substitution to check which of the possibilities in (1) are actually roots of $f(x)$.
(3) Now find all the roots of $f(x)$ (rational or otherwise).

Q3. Consider the polynomial

$$
p(x)=x^{7}-5 x^{4}-11 x^{3}+13 x+8
$$

(1) Use Descartes' Rules of Signs to determine the possible number of positive and negative roots of $p(x)$.
(2) Prove that $p(x)$ must have at least two roots which are not real numbers.

Q4. Consider the polynomial

$$
f(x)=x^{6}-8 x^{5}+28 x^{4}-58 x^{3}+77 x^{2}-50 x+50 .
$$

It is given to you that $1-2 i$ and $3+i$ are roots of $f(x)$. Find all the remaining roots of $f(x)$.

Q5. Let $u=\cos \left(\frac{\pi}{9}\right)$.
(1) Prove that $u$ is a root of the polynomial

$$
f(x)=8 x^{3}-6 x-1
$$

Hint: Use the formula

$$
\cos 3 \theta=\cos ^{3} \theta-3 \cos \theta \sin ^{2} \theta
$$

Recall that this formula appears in the exercises to Section 2.2, but you may use it here without proof.
(2) Find two more angles $\alpha$ and $\beta$ such that $u, \cos \alpha$ and $\cos \beta$ are all the roots of $f(x)$. Hint: The technique is broadly similar to how we find the $n$-th roots of a complex number.

