MATH 1210 Techniques of Classical and Linear Algebra

Assignment 3

Due on Monday, April 1

Attempt all questions and **show all your work.**

Completed assignment should be attached to the Honesty Declaration Form.

Only selected questions will be marked.

1. Find parametric and, if possible, symmetric equations of the line satisfying the following properties:

- passes through the point $P(1, 3, -2)$,
- is parallel to the plane $3x - 2z = 7$, and
- is perpendicular to the direction of the line $x = -1 + t$, $y = 2t$, $z = 5 - t$.

2. Determine whether the plane $x - 5y + 3z = -1$ intersects with the line

$$x = 2 - 3t,\ y = 4 + t,\ z = 1 - 2t,$$

and in case it does, find the point(s) of intersection.

3. Use Gauss-Jordan elimination to solve the following systems of linear equations. Clearly indicate every row operation you are performing.

(a)
$$\begin{aligned} 3x - 2y + z &= -1 \\ -x + 4y - 2z &= 2 \\ -2y + z &= -1 \\ x + 2y - z &= 1 \end{aligned}$$

(b)
$$\begin{aligned} 4x + y - z &= 3 \\ 2x - 5y - 3z &= 1 \\ -3x + 2y + 2z &= 0 \end{aligned}$$

4. Let $A = \begin{bmatrix} 6 & 1 & 10 & -11 & -2 \\ 7 & 2 & 0 & -2 & 1 \\ 2 & 1 & -6 & 5 & 2 \end{bmatrix}$.

(a) Find the rank of A.

(b) Find basic solutions of the linear system $AX = 0$.

5. Use Cramer's Rule to solve the following system of linear equations. **No marks will be given for any other method.**

$$\begin{aligned} -x + 3y + 4z &= 2 \\ 2y + 3z &= 1 \\ 5x + 4z &= -2 \end{aligned}$$

6. Consider the vectors $\mathbf{u}_1 = \langle 1, 2, 0\rangle$, $\mathbf{u}_2 = \langle -2, -5, 2\rangle$, and $\mathbf{u}_3 = \langle k, -6, k^2\rangle$, where k is a real number. Find all values of k for which vector $\mathbf{v} = \langle 3, 2, 1\rangle$ can be expressed as a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$.

7. Determine whether the vectors $\mathbf{u}_1 = \langle 5, 1, -1, 3\rangle$, $\mathbf{u}_2 = \langle 2, 0, 0, -1\rangle$, $\mathbf{u}_3 = \langle -3, -1, 2, 0\rangle$ are linearly dependent or linearly independent.