#### UNIVERSITY OF MANITOBA COURSE: MATH 1210 DATE & TIME: February 26, 2019, 17:45–19:00 MIDTERM EXAMINATION DURATION: 75 minutes

|            | I understand that cheating is a serious offence: |
|------------|--|
| Signature: | (In Ink)   |

# **INSTRUCTIONS**

- I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.
- II. This exam has a title page, 17 pages including this cover page and one blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.
- III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 60 points.
- IV. Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.
- V. Please do not call or e-mail your instructor to inquire about grades. They will be available shortly after they have been marked.
- VI. If the QR codes on your exam paper are deliberately defaced, your exam may not be marked.

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[8] 1. (a) Use mathematical induction to prove that

$$1 \cdot 3 + 2 \cdot 9 + 3 \cdot 27 + \ldots + n \cdot 3^{n} = \frac{3 + (2n - 1) \cdot 3^{n+1}}{4}$$

for every positive integer n.

[3] (b) Express the sum 
$$\frac{3}{20} - \frac{7}{19} + \frac{11}{18} - \frac{15}{17} + \ldots + \frac{35}{12}$$
 in sigma notation.

[7] 2. Write the following complex expression in **Cartesian form**.

$$\frac{64\,e^{\frac{2\pi}{3}i}}{(1-i)^{10}}$$

[7] 3. Find all cubic roots of

$$z = \frac{-3\sqrt{3}}{2} + \frac{9}{2}i$$

Write your answers in exponential form using the **principal value of the argument**.

[7] 4. Let  $P(x) = x^7 - kx^3 + 5$  where k is a complex number. Find the value of k if the remainder of division of P(x) by x - i is i.

5. Consider the polynomial

$$P(x) = 3x^7 + 11x^5 + 5x^4 + x^2 - 2x - 18.$$

[4] (a) Find all possible rational roots of P(x).

[5] (b) Use Descartes' Rules of Signs to prove that P(x) has at least 4 roots which are not real numbers.

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6. Consider the matrices 
$$A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & -1 & 4 \end{pmatrix}$$
,  $B = \begin{pmatrix} 3 & -1 \\ 1 & 5 \\ 6 & -2 \end{pmatrix}$ ,  $C = \begin{pmatrix} -3 & 1 \\ -1 & 6 \end{pmatrix}$ ,  $D = \begin{pmatrix} -1 & 7 & -10 \end{pmatrix}$  and  $E = \begin{pmatrix} 12 \\ -1 \\ -8 \end{pmatrix}$ .

[5]

(a) Indicate if each expression is defined or undefined by placing a check mark ( $\checkmark$ ) in the appropriate column. If it is defined, then indicate its size.

| EXPRESSION    | UNDEFINED | DEFINED | SIZE |
|---------------|-----------|---------|------|
| DE + D        |           |         |      |
| $(A^T + B)C$  |           |         |      |
| $(AE)^T + DB$ |           |         |      |

[4] (b) Evaluate  $B^T A^T - 2C$ .

- 7. Let  $\mathbf{u} = \langle 2, -2, 0 \rangle$  and  $\mathbf{v} = \langle -1, 0, -1 \rangle$ .
- [5] (a) Find the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .

[5] (b) Find the equation of the plane through the point (2, 7, 1) and parallel to both vectors **u** and **v**.

BLANK PAGE FOR ROUGH WORK (THIS PAGE WILL NOT BE MARKED.)