UNIVERSITY OF MANITOBA
COURSE: MATH 1210
DATE & TIME: April 18, 2020, 1:30–3:30 PM
Final EXAMINATION
DURATION: 2 hours

I understand that cheating is a serious offence:

Signature (*In Ink*): ____________________

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 23 pages including this cover page and one blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 100 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

V. Please do not call or e-mail your instructor to inquire about grades. They will be available shortly after they have been marked.

VI. If the QR codes on your exam paper are deliberately defaced, your exam may not be marked.

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[6] 1. Identities $\sum_{j=1}^{n} j = \frac{1}{2}[n(n+1)]$ and $\sum_{j=1}^{n} j^2 = \frac{1}{6}[n(n+1)(2n+1)]$ are given.
Use the identities to evaluate the sum

$$\sum_{j=5}^{12}(j-3)(2j-7)\,.$$

Your answer should be a single integer.

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2. Let $P(x) = 6x^4 + x^3 + 5x^2 + x - 1$.

[3] (a) Prove that $P(i) = 0$.

[7] (b) Use part (a) to find all zeros of $P(x)$.

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3. Let $P(3,5,-2)$ and $Q(2,1,6)$ be two points.

[4] (a) Find a vector of length 5 in the direction of $\overrightarrow{PQ}$.

[4] (b) Find parametric equations of the line that passes through the points P and Q.

[6] (c) Find an equation for the plane that contains the points P and Q and is parallel to the line $x = 3 + t$, $y = 9 + 3t$, $z = 2 - 7t$.

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4. Consider the homogeneous linear system

$$\begin{array}{rrrrcl} x_1 & +2x_2 & & +3x_4 & = & 0 \\ 3x_1 & +6x_2 & +2x_3 & -3x_4 & = & 0 \\ -x_1 & -2x_2 & +x_3 & -9x_4 & = & 0 \end{array}.$$

[2] (a) Determine the number of solutions of this system without solving it explicitly. You must give an adequate reason for your answer.

[7] (b) Find all solutions of this system using Gauss-Jordan elimination.

[3] (c) Find all basic solutions of this system.

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[6] 5. Let a be a nonzero number. Use Cramer's Rule to solve the following system of linear equations for z **only**. (No marks will be given for any other method.)

$$\begin{array}{rrrcr} x & +2y & +z & = & 1 \\ & 2ay & +(1+a)z & = & a \\ ax & & +z & = & -4 \end{array}.$$

6. Let $\mathbf{u}_1 = \langle 1, 0, 0, 2 \rangle$, $\mathbf{u}_2 = \langle 0, 1, -2, 3 \rangle$ and $\mathbf{u}_3 = \langle -1, 4, -8, 10 \rangle$.

[6] (a) Determine if the vectors $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ are linearly dependent or linearly independent.

[8] (b) Find all value(s) of a such that the vector $\mathbf{v} = \langle -1, 2, -4, a \rangle$ can be expressed as a linear combination of the vectors $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$.

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7. Let $A = \begin{pmatrix} -3 & -1 & 3 \\ -1 & 0 & 1 \\ -2 & 0 & 1 \end{pmatrix}$.

[7] (a) Use the direct method (i.e., elementary row-operations) to find the inverse of the matrix A.

[3] (b) Solve the linear system of equations $(A^{-1})^T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

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[6] 8. Let $\mathbf{a} = \langle 5, -1, 2 \rangle$. Define a transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ by the formula

$$T(\mathbf{v}) = \mathbf{v} \times \mathbf{a}.$$

You may assume without proof that T is a linear transformation. Find the matrix associated to T.

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9. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the linear transformation given by

$$\begin{aligned} v_1' &= 7\,v_1 - v_2 \\ v_2' &= 3\,v_1 + 2\,v_2 \end{aligned}$$

and let $\mathbf{u} = \langle 1, 1 \rangle$.

[2] (a) Find $T(\mathbf{u})$.

[8] (b) Find a vector $\mathbf{v} = \langle a, b \rangle$ such that $T(2\mathbf{u} - \mathbf{v}) = -3\mathbf{v}$.

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10. Consider the matrix $B = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix}$.

[5] (a) Given that -2 is an eigenvalue of the matrix B, find the remaining eigenvalues of B.

[7] (b) Find all the eigenvectors corresponding to the eigenvalue -2.