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[6] 1. Use the formulas (valid for all $t \in \mathbb{R}$),

$$\cos(t) = \frac{1}{2}\left(e^{it} + e^{-it}\right), \qquad \sin(t) = \frac{i}{2}\left(e^{-it} - e^{it}\right), \tag{1}$$

to calculate

$$2\sin(\pi t)\cos(\pi t).$$

Any solution in which the formulas in equation (1) are not used will be given a zero. Your answer must be of the form $\sin(\lambda t)$ for some $\lambda \in \mathbb{R}$. (You don't need to show that the formulas in Eq. (1) are valid.)

[9] 2. Let A be an upper triangular matrix. Use the principle of induction to prove that for all positive integers n, A^n is upper triangular. You may use (without proving it) the fact that if B and C are upper triangular matrices of the same size, then BC is also upper triangular.

[6] 3. Given that $z_1 = 3 + i$ is a zero of $f(x) = x^4 - 6x^3 + 13x^2 - 18x + 30$, find all zeros of $f(x)$.

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4. Consider the matrix

$$B = \begin{pmatrix} 0 & -a & -b \\ 0 & 0 & -c \\ 0 & 0 & 0 \end{pmatrix}$$

[6] (a) Calculate B^n for all integers $n \geqslant 2$. Your answer should be presented in the form:

$$B^n = \begin{cases} ... & \text{if } n = 2 \\ ... & \text{if } n = 3 \\ ... & \text{if } n \geqslant 4 \end{cases}$$

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[0] (b) **BONUS: MAX 4 MARKS.** Calculate the matrix D given by the formula

$$D = \mathrm{I}_3 + \sum_{n=1}^{5} B^n.$$

Let A be the matrix $A = \mathrm{I}_3 - B$, then calculate AD.