

UNIVERSITY OF MANITOBA
Term Test 2A

COURSE: MATH 1210

DATE & TIME: Mar 5, 50 Minutes

DURATION: IN CLASS

EXAMINER: Borgersen/Kristel

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- [6] 1. Use the formulas (valid for all $t \in \mathbb{R}$),

$$\cos(t) = \frac{1}{2} (e^{it} + e^{-it}), \quad \sin(t) = \frac{i}{2} (e^{-it} - e^{it}), \quad (1)$$

to calculate

$$2 \sin(\pi t) \cos(\pi t).$$

Any solution in which the formulas in equation (1) are not used will be given a zero. Your answer must be of the form $\sin(\lambda t)$ for some $\lambda \in \mathbb{R}$. (You don't need to show that the formulas in Eq. (1) are valid.)

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- [9] 2. Let A be an upper triangular matrix. Use the principle of induction to prove that for all positive integers n , A^n is upper triangular. You may use (without proving it) the fact that if B and C are upper triangular matrices of the same size, then BC is also upper triangular.

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- [6] 3. Given that $z_1 = 3 + i$ is a zero of $f(x) = x^4 - 6x^3 + 13x^2 - 18x + 30$, find all zeros of $f(x)$.

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4. Consider the matrix

$$B = \begin{pmatrix} 0 & -a & -b \\ 0 & 0 & -c \\ 0 & 0 & 0 \end{pmatrix}$$

[6] (a) Calculate B^n for all integers $n \geq 2$. Your answer should be presented in the form:

$$B^n = \begin{cases} \dots & \text{if } n = 2 \\ \dots & \text{if } n = 3 \\ \dots & \text{if } n \geq 4 \end{cases}$$

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- [0] (b) **BONUS: MAX 4 MARKS.** Calculate the matrix D given by the formula

$$D = I_3 + \sum_{n=1}^5 B^n.$$

Let A be the matrix $A = I_3 - B$, then calculate AD .