UNIVERSITY OF MANITOBA

First Assignment

COURSE: MATH 1210 DUE DATE: Feb 18, 5 PM

RELEASE DATE: Feb 8, 5 PM EXAMINER: Comicheo/Moghaddam/Shepelska

---

Attempt all questions and show all your work. Some or all questions will be marked. Simplify your answers as much as possible.

1. Use mathematical induction to prove each of the following statements:
   (a) $8n^3 + 12n^2 - 2n - 3$ is divisible by 3, for every integer $n \geq 1$.
   (b) $(2n) + (2n+1) + (2n+2) + \cdots + (2n+n) = \dfrac{5n(n+1)}{2}$ for every integer $n \geq 3$;
   (c) $\displaystyle\sum_{i=1}^{2^n} \frac{1}{i} > \frac{n}{2}$ for every integer $n \geq 1$.

2. Write the sum below in sigma notation:

$$-\sqrt{2} + \frac{2\sqrt{3}}{3} - \frac{3(2)}{5} + \frac{4\sqrt{5}}{7} - \frac{5\sqrt{6}}{9} + \cdots - \frac{35(6)}{69}$$

3. Use the identities

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \text{and} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

   to compute the sum:

$$9(10)(11) + 10(11)(12) + 11(12)(13) + \cdots + 21(22)(23)$$

   Show all the steps. It is not necessary to simplify your final answer to a single number: for example, expressions like $\left(\frac{22 \cdot 23}{2}\right)^2$, or 8^{80}, do not need further simplification.

---

4. Write the following complex numbers in exponential form. The principal value of the argument should be of the form $\frac{a\pi}{b}$ where a and b are integers and $b \neq 0$. Show all the steps.
   (a) $\overline{-\left(2e^{i\pi/6}\right)^{11}}$
   (b) $\dfrac{-3-3\sqrt{3}i}{5+5i}$
   (c) $\dfrac{1+\sqrt{3}+(\sqrt{3}-1)i}{1+\sqrt{3}+(1-\sqrt{3})i}$
   (d) $(-\sqrt{21}+\sqrt{7}i)^{101}$

5. Find all the solutions of the equation $x^6+8=4x^3$. Show all the steps. Write the roots in exponential form and use the principal value of their arguments.