UNIVERSITY OF MANITOBA
Second Assignment

COURSE: MATH 1210 DUE DATE: March 11, 5 PM
RELEASE DATE: March 1, 8:30 AM EXAMINER: Comicheo/Moghaddam/Shepelska

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Attempt all questions and show all your work. Some or all questions will be marked. Simplify your answers as much as possible.

1. Let h be a real number and $P(x) = x^5 - 5x^3 + (h-4)x^2 - h$.
   (a) Prove that $x = -1$ is a root of $P(x)$ regardless of the value of h.
   (b) Find all values of h for which the multiplicity of the root $x = -1$ of $P(x)$ is greater than one. (Hint: express $P(x)$ as $P(x) = (x+1)Q(x)$.)

2. Let $P(x) = x^5 + 3x^4 - x^3 - 10x^2 - 14x - 4$. Given that $(-1+i)$ is a zero of $P(x)$, find all zeros of $P(x)$.

3. Consider the equation $4x^8 - 6x^7 - 2x^5 + 4x^3 + x^2 - 9x = 10$.
   (a) Find the possible number of positive and the possible number of negative real solutions of this equation.
   (b) Prove that the above equation has at least two non-real solutions.

4. Note that the Bounds Theorem holds for the polynomials with complex coefficients and complex roots if one interprets $|w|$ as the modulus of w.
   (a) Apply the Bounds Theorem to find the upper bound for moduli of the roots of the polynomial
   $$P(x) = (3-4i)x^4 - (7+7i)x^3 - 4x^2 + (8+6i)x + 9i.$$
   Show all your work.
   (b) Use the result of (a) to prove that $x = \sqrt{7} - 2i$ cannot be a root of $P(x)$.

5. Let $P(x) = 6x^5 - x^4 + 4x^3 - 13x^2 + 4$.
   (a) Use the Rational Roots Theorem to find all possible rational roots of $P(x)$.
   (b) Find all roots of $P(x)$.

6. Consider the matrices
$$A = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & -1 \\ 3 & 4 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 & -5 \\ -1 & 4 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & 2 & 1 \\ -2 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}.$$

In parts (a)-(e) find the specified matrix when possible. If not possible, explain why.

(a) $3A - 4B$ (b) A^3 (c) BAD (d) ADB (e) $3AC - AB^T$

(f) Find the matrix X that satisfies the equation $2X^T + I_2 = CB$.

(g) Find the dimensions of a matrix Y that would allow for the product YDC^TY to be defined.