
Attempt all questions and show your work.

[8] 1. Let $P(x) = 4x^4 - x^3 - x^2 - x - 5$. Find all the roots of $P(x)$ provided that $-i$ is one of them.

2. Consider the plane $\Pi : 2x + 3y + z = 9$ and the lines:

$$\begin{aligned} \mathcal{L}_1 &: \; x = 1, \; y - 1 = -t, z = 1 - 3t \\ \mathcal{L}_2 &: \; x = 2s + 1, \; y = 1 - 3s, z = 1\,. \end{aligned}$$

[3] (a) Find the intersection point of the line $\mathcal{L}_1$ and the plane Π.

[7] (b) Find a point-normal equation of the plane Π_1 containing two intersecting lines $\mathcal{L}_1$ and $\mathcal{L}_2$.

[5] 3. Consider the system of linear equations

$$\begin{aligned} x_1 + 2x_2 \quad -3x_3 + 2x_4 \quad +3x_5 &= 0 \\ x_1 + 2x_2 \quad -3x_3 + 5x_4 \quad +4x_5 &= 0\,. \\ 3x_1 + 6x_2 \quad -9x_3 + 7x_4 \quad +9x_5 &= 0 \end{aligned}$$

Without solving predict the number of solutions of this system. Explain.

[7] 4. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ -2 & 1 & -1 \end{bmatrix}$. Given that $|AB^{-1} + 4I| = 23$, find determinant of the matrix

$$A^{10}B^{-10} + 4A^9B^{-9}\,.$$

[9] 5. Let $\mathbf{u} =< 1, 0, a, 0 >$, $\mathbf{v} =< 0, 1, a^2 - 4, a >$ and $\mathbf{w} =< 0, 1, 0, a >$ where a is a real number. Find **all** values of a for which $\{\mathbf{u}\,, \mathbf{v}\,, \mathbf{w}\}$ is a linearly dependent set. Show your work.

[6] 6. Let k be a real number and consider the linear system

$$\begin{cases} kx + 2z &= 4 \\ 2x - y &= -2 \\ 3y + z &= 1 \end{cases}\,.$$

Use Cramer's rule to find all values of k for which $x = -1$.

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7. Consider the matrix $A = \begin{pmatrix} -1 & 3 & 2 \\ 0 & 1 & 1 \\ 2 & -6 & -3 \end{pmatrix}$.

[7] (a) Find the inverse of A by direct method using elementary row operations.

[3] (b) Solve the linear system $2A^T \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 6 \\ -8 \\ 0 \end{pmatrix} = \mathbf{0}$, where $\mathbf{0}$ is the zero matrix of the appropriate size.

8. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T\big(\langle 3, 0\rangle\big) = \langle 6, 9\rangle$ and $T\big(\langle 0, -1\rangle\big) = \langle 2, -5\rangle$.

[6] (a) Find the matrix associated with T.

[2] (b) Find the image of $\langle -1, 4\rangle$ under T.

9. Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

[5] (a) Show that the only eigenvalue of the matrix A is $\lambda = 3$ by solving the characteristic equation.

[7] (b) Find all eigenvectors corresponding to eigenvalue $\lambda = 3$.