

MATH 1210 Midterm Exam Winter 2022

Date and Time: March 14 at 6:00 PM

Duration: 75+15 minutes

Attempt all questions and show your work. Simplify your answers as much as possible.

1. (8 points) Use mathematical induction to show that the following identity holds for every integer $n \geq 2$.

$$1 + 3 + 5 + \cdots + (4n - 1) = 4n^2$$

2. (5 points) Write the following sum in sigma notation. Your index of summation should start from 1.

$$\frac{3}{(5)(9)} - \frac{3^2}{(6)(10)} + \frac{3^3}{(7)(11)} - \frac{3^4}{(8)(12)} + \cdots - \frac{3^{10}}{(14)(18)}$$

3. (10 points) Simplify the expression below and then write it in Cartesian form.

$$\frac{\left(\sqrt{3} - e^{-\frac{\pi}{6}i}\right)^6}{2 + i}$$

4. Let $P(x) = x^3 + (3 - 2i)x^2 + (3 - 6i)x - 6i$.

(a) (3 points) Verify that $x = 2i$ is a root of $P(x)$.

(b) (7 points) Use the result of part (a) to find all roots of $P(x)$.

5. Let $P(x) = 2x^5 - 3x^4 + 4x^2 - x - 4$.

(a) (4 points) Use Rational Roots Theorem to find all possible rational roots of $P(x)$.

(b) (4 points) Apply Bounds Theorem to $P(x)$ and use it to eliminate some possible roots from the list obtained in (a).

You are **not** asked to find zeros of $P(x)$.

6. Consider the matrices

$$A = \begin{bmatrix} -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 5 & -1 \\ 1 & 0 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -2 \\ 1 & 2 \\ -1 & 3 \end{bmatrix}.$$

(a) (6 points) Find the matrix $2AA^T - BC$.

(b) (3 points) Suppose that E is a matrix such that the expression E^2B is defined. Find the size (dimensions) of E .

7. Consider three vectors $\mathbf{u} = \langle -1, 2, -2 \rangle$, $\mathbf{v} = \langle 1, 4, -1 \rangle$ and $\mathbf{w} = \langle a, a + 3, b \rangle$, where a and b are real numbers.

(a) (5 points) Find the angle between \mathbf{u} and \mathbf{v} .

(b) (5 points) Find values of a and b such that \mathbf{w} is perpendicular to both \mathbf{u} and \mathbf{v} .