MATH 1210 Assignment 1 Winter 2023 **Due date: February 6**

Attempt **all** questions and show all your work. Selected number of questions will be marked.

Attach to Honesty Declaration Form

1. Use mathematical induction on integer n to prove each of the following:

   (a) $\dfrac{1}{3\cdot 2\cdot 3}+\dfrac{1}{3\cdot 3\cdot 4}+\dfrac{1}{3\cdot 4\cdot 5}+\cdots+\dfrac{1}{3n(n+1)}=\dfrac{n-1}{6(n+1)}$ for $n \geq 2$;

   (b) $11^{2n}+11^{n}-2$ is divisible by 10 for $n \geq 1$.

   (c) $(1-\dfrac{3}{4})(1-\dfrac{5}{8})(1-\dfrac{7}{12})\cdots(1-\dfrac{2n+1}{4n}) = \dfrac{(2n)!}{2^{3n}(n!)^2}$, for $n \geq 1$. (You are **not** allowed to use any other method.)

2. Identities $\sum_{k=1}^{m} k = \dfrac{1}{2}\,[m(m+1)]$ and $\sum_{k=1}^{m} k^2 = \dfrac{1}{6}\,[m(m+1)(2m+1)]$ are given.

   (a) First write the sum $1^2+4^2+7^2+10^2+\cdots+(6n+1)^2$ in sigma notation and then use the identities to prove that

   $$1^2+4^2+7^2+10^2+\cdots+(6n+1)^2 = (2n+1)(12n^2+9n+1)\,.$$

   (b) Use the identities to evaluate the sum $\sum_{j=16}^{27}[\,(3j-45)^2 - 7\,]$.

3. Express each of the following in simplified Cartesian form.

   (a) $\dfrac{(\sqrt{2}-\sqrt{6}\,i)^{34}}{(2\sqrt{2}+2\sqrt{6}\,i)^{20}}$;

   (b) $\dfrac{1}{2^{10}}\Big(\,i^{27}(1-i)^{20} + (-\sqrt{3}+i)^{11}\,\Big)$.

4. Find the complex number z, in Cartesian form, such that it satisfies the equation

   $$(1-\sqrt{3}\,i)^5 + \overline{(\sqrt{3}-i\,)^5}\,z = 32\,(1+\sqrt{3}\,i)\,.$$

5. Find all complex solutions of the equation $\dfrac{1}{3}z^5 - 81(\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i) = 0$. Write the roots in exponential form and use principal value of their arguments.