MATH 1210 Assignment 2 Winter 2023

Due date: February 28, 5:00 PM

Attempt all questions and show all your work. Some or all questions will be marked.

- 1. Let $P(x) = 2x^3 2ix^2 + (-3 + 2i)x 1 + i$.
 - (a) Verify that -1 + i is a root of P(x).
 - (b) Find all roots of P(x).
- 2. Let k be a real number and $P(x) = 9x^4 + 3kx^3 2x^2 (k^2 3)x 2$.
 - (a) Find all values of k such that (3x 2) is a factor of P(x).
 - (b) For k = 2, solve the equation P(x) = 0.
- 3. Let $P(x) = 10x^7 2x^6 + 5x^5 + x^3 7x^2 1$.
 - (a) Find the possible number of positive real zeros of P(x).
 - (b) Use Descartes' Rules of Signs to show that $1 \sqrt{2}$ cannot be a zero of P(x).
 - (c) Given that the Bounds Theorem holds for the complex roots if one interprets $|x_0|$ as the modulus of x_0 , find an upper bound on moduli of the roots of P(x).
 - (d) Use the result of part (c) to prove that $\frac{4\sqrt{2}}{5} \frac{7}{5}i$ cannot be a root of P(x).
- 4. Let $P(x) = 10x^5 35x^4 + 22x^3 + 13x^2 + 4x + 4$.
 - (a) Use the Rational Roots Theorem to find all possible rational roots of P(x).
 - (b) Find all roots of P(x).
- 5. Consider the matrices

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ -2 & 3 \\ 4 & 2 \end{bmatrix}, \qquad C = \begin{bmatrix} 3 & 1 & 2 \\ -4 & 0 & 5 \end{bmatrix}, \qquad D = \begin{bmatrix} -2 & 4 & 0 \\ 3 & 0 & 1 \\ 2 & -1 & 6 \end{bmatrix}.$$

For each of the following expressions, determine if it is defined. If yes, evaluate it. If no, explain why. (a) $(2B - 3C^T)A$ (b) ACD (c) DB + CD

(d) Find the matrix X that satisfies the equation $(A - I_2)^2 X - 2X(A - I_2)^2 = 2A$. (Hint: let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.)

6. Let $\mathbf{u} = \langle 2, 1, -1 \rangle$ and $\mathbf{v} = \langle 4, 1, -3 \rangle$. Find each of the following.

- (a) the angle between \mathbf{u} and $\mathbf{u} \mathbf{v}$
- (b) the vector of length 5 in the direction opposite to \mathbf{v}
- 7. Consider the plane $\pi : x 2y = 4$, the line $\ell : x = 3 t$, y = -1 + 4t, z = 2 2t, and the points P(2, 4, -1) and Q(3, 1, 0).
 - (a) Find an equation of the plane that passes through the points P and Q and is perpendicular to the plane π .
 - (b) Write parametric equations of the line passing through the points P and Q.
 - (c) Does the line found in part (b) intersect with the line ℓ ? If yes, find the intersection point.