Attempt all questions and show all your work. Some or all questions will be marked.

1. Let $P(x)=2 x^{3}-2 i x^{2}+(-3+2 i) x-1+i$.
(a) Verify that $-1+i$ is a root of $P(x)$.
(b) Find all roots of $P(x)$.
2. Let $k$ be a real number and $P(x)=9 x^{4}+3 k x^{3}-2 x^{2}-\left(k^{2}-3\right) x-2$.
(a) Find all values of $k$ such that $(3 x-2)$ is a factor of $P(x)$.
(b) For $k=2$, solve the equation $P(x)=0$.
3. Let $P(x)=10 x^{7}-2 x^{6}+5 x^{5}+x^{3}-7 x^{2}-1$.
(a) Find the possible number of positive real zeros of $P(x)$.
(b) Use Descartes' Rules of Signs to show that $1-\sqrt{2}$ cannot be a zero of $P(x)$.
(c) Given that the Bounds Theorem holds for the complex roots if one interprets $\left|x_{0}\right|$ as the modulus of $x_{0}$, find an upper bound on moduli of the roots of $P(x)$.
(d) Use the result of part (c) to prove that $\frac{4 \sqrt{2}}{5}-\frac{7}{5} i$ cannot be a root of $P(x)$.
4. Let $P(x)=10 x^{5}-35 x^{4}+22 x^{3}+13 x^{2}+4 x+4$.
(a) Use the Rational Roots Theorem to find all possible rational roots of $P(x)$.
(b) Find all roots of $P(x)$.
5. Consider the matrices

$$
A=\left[\begin{array}{rr}
2 & -1 \\
1 & 2
\end{array}\right], \quad B=\left[\begin{array}{rr}
1 & 0 \\
-2 & 3 \\
4 & 2
\end{array}\right], \quad C=\left[\begin{array}{rrr}
3 & 1 & 2 \\
-4 & 0 & 5
\end{array}\right], \quad D=\left[\begin{array}{rrr}
-2 & 4 & 0 \\
3 & 0 & 1 \\
2 & -1 & 6
\end{array}\right]
$$

For each of the following expressions, determine if it is defined. If yes, evaluate it. If no, explain why.
(a) $\left(2 B-3 C^{T}\right) A$
(b) $A C D$
(c) $D B+C D$
(d) Find the matrix $X$ that satisfies the equation $\left(A-I_{2}\right)^{2} X-2 X\left(A-I_{2}\right)^{2}=2 A$.
(Hint: let $X=\left[\begin{array}{ll}a & b \\ c & d\end{array}\right]$.)
6. Let $\mathbf{u}=\langle 2,1,-1\rangle$ and $\mathbf{v}=\langle 4,1,-3\rangle$. Find each of the following.
(a) the angle between $\mathbf{u}$ and $\mathbf{u}-\mathbf{v}$
(b) the vector of length 5 in the direction opposite to $\mathbf{v}$
7. Consider the plane $\pi: x-2 y=4$, the line $\ell: x=3-t, y=-1+4 t, z=2-2 t$, and the points $P(2,4,-1)$ and $Q(3,1,0)$.
(a) Find an equation of the plane that passes through the points $P$ and $Q$ and is perpendicular to the plane $\pi$.
(b) Write parametric equations of the line passing through the points $P$ and $Q$.
(c) Does the line found in part (b) intersect with the line $\ell$ ? If yes, find the intersection point.

